SECTIONA: MECHANICS

CHAPTER1: DIMENSIONS OF A PHYSICAL QUANTITY

1.1.0: Fundamental quantities

These are quantities which can't be expressed in terms of any other quantities by using any mathematical equation. E.g.

Mass - M | Length - L | Time- T

1.1.1: Derived quantities

These are quantities which can be expressed in terms of the fundamental quantities of mass, length, and time e.g.

i) Pressure
ii) Acceleration
iii) Momentum
iv) Density

1.1.2: DIMENSIONS OF A PHYSICAL QUANTITY

This refers to the way a physical quantity is related to the three fundamental quantities of length, mass and time.

Or It refers to the power to which fundamental quantities are raised.

Symbol of dimensions is [ ]

Examples

[Area] $= L^2$

[Volume]$= L^3$

$[Density] = \frac{[Mass]}{[Volume]} = \frac{M}{L^3}$ =ML^{-3}

[Velocity]= $\frac{[Displacement]}{[Time]} = \frac{L}{T} = LT^{-1}$

[Acceleration]= $\frac{[Change\ in\ Velocity]}{[Time]} = \frac{LT^{-1}}{T} = LT^{-2}$

[Momentum]=[Mass][Velocity]=MLT^{-1}

[Weight]=[Mass][Gravitational acceleration]=MLT^{-2}

[Force]=[Mass][Acceleration]= MLT^{-2}

[Pressure]= $\frac{[Force]}{[Area]} = \frac{MLT^{-2}}{L^2}$ =M$L^{-1}T^{-2}$

NB. Dimension less quantity has no dimensions and is described by a number which is independent of a unit of measurement chosen for the primary quantities

Examples of dimension less quantities

- Refractive index
- strain
- relative density
- all constants such as 2π, 2, π, 4 π, .

They are always given a dimension of one, (1)

1.1.3: USES OF DIMENSIONS

1. Used to check the validity of the equation or check whether the equation is dimensionally consistent or correct.
2. Used to derive equations

a) Checking validity of equations (dimensional homogenity)

When the dimensions on the L-H-S of the equations are equal to the dimensions on the R-H-S, then the equation is said to be dimensionally consistent.

Examples

1. The velocity V of a wave along a flat string is given by $V = \sqrt{\frac{TL}{M}}$

T - Tension in the string
L - Length of the string
M - Mass of the string

Show that the formula is dimensionally correct.

Solution

$$V = \sqrt{\frac{TL}{M}}$$

L.H.S $[V] = LT^{-1}$

R.H.S $\left[\sqrt{\frac{TL}{M}}\right] = \left[\left(\frac{TL}{M}\right)^{\frac{1}{2}}\right] = \left(\frac{[T][L]}{[M]}\right)^{\frac{1}{2}}$

Tension (T) is a force therefore takes the dimensions of force.

$\left(\frac{MLT^{-2}L}{M}\right)^{\frac{1}{2}} = (L^2T^{-2})^{\frac{1}{2}}$

$= L^{2x\frac{1}{2}}T^{-2x\frac{1}{2}}$

$= LT^{-1}$

L.H.S = R.H.S

Since dimension on left are equal to dimensions on right then its correct

2. The period T, of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ Show that the equation is dimensionally correct.

Where 2π = dimension less constant

l = length of pendulum

g = Acceleration due to gravity

Solution

L.H.S $[T] = T$

R.HS = $\left[2\pi\sqrt{\frac{l}{g}}\right] = \left[2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}}\right] = [2\pi]\left(\frac{[l]}{[g]}\right)^{\frac{1}{2}}$

$= \left(\frac{L}{LT^{-2}}\right)^{\frac{1}{2}} = (T^2)^{1/2} = T$

Since the dimensions on the L.H.S are equal to the dimensions on the R.H.S then the equation is dimensionally consistent.

NB: Dimensions cannot be added or subtracted but for any equation to be added or subtracted then they must have the same dimensions.

Example

Show that the equation $v^2 = u^2 + 2as$ is dimensionally correct.

Solution

L.H.S $[v^2] = (LT^{-1})^2 = L^2T^{-2}$

R.H.S = $[u^2]$ = [2as]

$= (LT^{-1})^2 = LT^{-2}L$

$= L^2T^{-2} = L^2T^{-2}$

Since dimensions on the L.H.S are equal to dimensions on the R.H.S then the equation is dimensionally correct.

Exercise

1. Show that the following equations are dimensionally consistent wher symbols have their usual meanings

i) $S = ut + \frac{1}{2}at^2$ ii) $v = ut + at$ iii) $Ft = mv - mu$

2. The frequency f of vibration of the drop of a liquid depends on surface tension, γ of the drop, its density, ρ and radius r of the drop. Show that $f = k\sqrt{\frac{\gamma}{\rho r^2}}$ where k is a non= dimensional constant

b) Deriving equations (dimensional analysis)

The method of dimension analysis is used to obtain an equation which is relating to relevant variables

Examples

1. Assume that the period (T) depend on the following
   i) Mass (m) of the bob
   ii) Length (l) of the pendulum
   iii) Acceleration due to gravity (g)

   Derive the relation between T, m, l, g

   Solution

$T \propto m^x l^y g^z$

$T = K m^x l^y g^z$ x

Where K is a constant

If it's dimensionally consistent then

$[T] = [K][m]^x[l]^y[g]^z$

$T = M^x L^y (LT^{-2})^z$

$M^0L^0T = M^xL^yL^ZT^{-2Z}$

$M^0L^0T = M^xL^{y+Z}T^{-2Z}$

Powers of M; $x = 0$..............1

poweers of L; $y + z = 0$.........2

powers of T; $-2z = 1$ 3

$z = \frac{-1}{2}$

Put into (2); $y + \frac{-1}{2} = 0$

$y = \frac{1}{2}$

$x = 0, y = \frac{1}{2}, z = \frac{-1}{2}$

Since $T = K m^x l^y g^z$

$T = K m^0 l^{\frac{1}{2}} g^{\frac{-1}{2}}$

$T = K\frac{l^{\frac{1}{2}}}{g^{\frac{1}{2}}}$

$T = K\sqrt{\frac{l}{g}}$

2. Use dimensional analysis to show how the velocity of transverse vibrations of a stretched string depends on its length (l) mass (m) and the tension force (F) in the string.

   solution

$V \propto l^x m^y F^z$

$V = K l^x m^y F^z$

$[V] = [K l^x m^y F^z]$

$[V] = [K][l^x][m^y][F^z]$

$LT^{-1} = L^x M^y (MLT^{-2})^z$

since $[K] = 1$

$MLT^{-1} = L^{x+z}M^{y+z}T^{-2z}$

Powers of M; $y + z = 0$..........(1)

Powers of L; M $x + z = 1$......(2)

Powers of T; $-2z = -1$...........(3)

$Z = \frac{1}{2}$

Put into (1); $y + z = 0$

$y + \frac{1}{2} = 0$

$y = \frac{-1}{2}$

Also for equation(2); $x + z = 1$

$x + \frac{1}{2} = 1 \therefore x = \frac{1}{2}$

but $V = K l^x m^y F^z$

$V = K l^{\frac{1}{2}} m^{\frac{-1}{2}} F^{\frac{1}{2}}$

$V = K\sqrt{\frac{l F}{m}}$

3. The viscous force (F) on a small sphere of radius (a) falling through a liquid of coefficient of viscosity ŋ with a velocity V given by $F = K a^x \eta^y V^z$

   Use the method of dimensions to find the values of x, y, z (5marks)

   Solution

$[\eta] = \frac{[Force]}{[Area]x[\ vel\ gradient]}$

[F] = MLT^{-2} and [A] = L^2

[Velocity gradient] = $\frac{[V_2 - V_1]}{[l]}$

[Velocity gradient]= $\frac{LT^{-1}}{L} = T^{-1}$

$[\eta] = \frac{MLT^{-2}}{L^2T^{-1}}$

[ŋ] = M $L^{-1}T^{-1}$

[F]=[K]$[a^x][\eta^y][V^z]$

$MLT^{-2} = L^x (ML^{-1}T^{-1})^y (LT^{-1})^z$

$MLT^{-2} = M^y L^{x+z-y} T^{-y-z}$

For M : $y = 1$..........(1)

For L: $x + z - y = 1$.......(2)

For T: $-y - z = -2$........(3)

Put (1) into (3)

$-y - z = -2$

$-1 - z = -2$

$Z = 1$

Put into equation(2)

$x + z - y = 1$

$x + 1 - 1 = 1$

$x = 1$

F=$K a^x \eta^y V^z$

F = K a ŋ V

UNEB 2016 NO 1 (a)

(i) Define dimensions of a physical quantity. (01mark)

(ii) In the gas equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Where P= pressure, V= volume, T=absolute temperature, and R= gas constant. What are the dimensions of the constants a and b. (04marks)

UNEB 2010 NO 4 (d)

The velocity V of a wave in a material of young modulus E and density ρ is given by $V = \sqrt{\left(\frac{E}{\rho}\right)}$

Shows that the relationship is dimensionally correct (03 marks)

UNEB2009 No 3b

A cylindrical vessel of cross sectional area, A contains air of volume V, at pressure p trapped by frictionless air tight piston of mass, M. The piston is pushed down and released.

i) If the piston oscillates with simple harmonic motion, shows that its frequency f is given $f = \frac{A}{2\pi}\sqrt{\frac{\mathrm{p}}{\mathrm{MV}}}$ (06 marks)

ii) Show that the expression for f in b(i) is dimensionally correct (03 marks)

UNEB 2005 No1 b

The equation for the volume V of a liquid flowing through a pipe in time t under a steady flow is given by $\frac{V}{t} = \frac{\pi r^4 P}{8\,\eta l}$

Where r = radius of the pipe η = coefficient of viscosity of the liquid

P =pressure difference between the 2 ends

l =length of the pipe

Show that the equation is dimensionally consistent (3mks)

UNEB2003 No 1(a)

Distinguish between fundamental and derived physical quantities. Give two examples of each (04marks)

UNEB2002 No1

a) i)What is meant by the dimension of a physical quantity (01mark)

ii)For a stream line flow of a non-viscous, in compressible fluid, the pressure P at a point is related to the width h and the velocity v by the equation.

(P -a) = ρg(h-b) + ½ ρ (v^2-d) where a, b and d are constant and ρ is the density of the fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent, find the dimensions of a, b and d (03 marks)

Solution

NB: We only add and subtract quantities which have the same dimensions.

$(P - a) = \rho g(h - b) + ½\,\rho\,(v^2 - d)$

LHS: $[P] = [a]$

$$[P] = \frac{[Force]}{[Area]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$[a] = ML^{-1}T^{-2}$

On the RHS: $[h] = [b]$

$[b] = L$

$[v^2] = [d]$

$(LT^{-1})^2 = [d]$

$[d] = L^2T^{-2}$

UNEB 2001 No 2 b

The velocity V of sound travelling along a rod made of a material of young's modulus y and density ρ is given by V=$\sqrt{\frac{Y}{\rho}}$ Show that the formula is dimensionally consistent (03 mks)

UNEB 1997 No 1

a) i) What is meant by dimensions of a physical quantity (1mk)

ii)The centripetal force required to keep a body of mass m moving it a circular path of radius r is given by $F = \frac{m\,v^2}{r}$ show that the formula is dimensionally consistent. (04 marks)

CHAPTER 2: MOTION

2.1.0: LINEAR MOTION

This is motion is a straight line

Distance

This is the length between 2 fixed point

Displacement

This is the distance covered in a specific direction

Speed

This is the rate of change of distance with time

OR It is the distance covered by an object per unit time.

The SI unit of speed is ms^{-1}

Velocity

It is the rate of change of displacement with time

OR It is the distance covered per unit time in a specific direction

The SI unit of velocity is ms^{-1}

Uniform velocity

Is the velocity of a body which covers equal displacement in equal time intervals.

Acceleration

It is the rate of change of velocity with time

It SI unit is ms^{-2}

Acceleration $= \frac{change\ in\ velocity}{time}$ | $a = \frac{v-u}{t}$

Uniform acceleration

Constant rate of change of velocity.

Equations of uniform acceleration

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in a time t, then from definition of acceleration

$a = \frac{v-u}{t}$ | $at = v - u$ | $\boxed{v = u + a\,t}$ 1

2nd equation

Suppose an object with velocity u moves with uniform acceleration for a time t and attains a velocity v, the distance s travelled by the object is given by S = average velocity x time

$S = \left(\frac{v+u}{2}\right)t$

But $v = u + at$

$S = \frac{(u + at + u)}{2}t$

$S = \frac{(2u + at)t}{2}$

$S = \frac{2ut + at^2}{2}$

$\boxed{S = ut + \frac{1}{2}at^2}$ 2

3rd equation

S = average velocity x time

$S = \left(\frac{v+u}{2}\right)t$

But $t = \frac{v-u}{a}$

$S = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$

S $= \frac{(v+u)(v-u)}{2a}$

S $= \frac{v^2-u^2}{2a}$

2as $= v^2 - u^2$

$\boxed{v^2 = u^2 + 2as}$ 3

Note

- The three equations apply only to uniformly accelerated motion
- When the object starts from rest then (u=0m/s) and when it comes to rest (v=0m/s)

- The acceleration can be positive or negative. When its negative, then it known as a retardation or deceleration

Examples

1) A car moving with a velocity of $10ms^{-1}$ accelerates uniformly at $1ms^{-2}$ until it reaches a velocity of $15ms^{-1}$. Calculate,
   i) Time taken
   ii) Distance traveled during the acceleration
   iii) The velocity reached 100m from the place where acceleration began.

Solution

i) $v = u + at$
u=10m/s, a=1m/s, v=$15ms^{-1}$,
$15 = 10 + t$
$t = 5s$
ii) $v^2 = u^2 + 2as$

$15^2 = 10^2 + 2x1xs$
$225 = 100 + 2s$
$S = 62.5m$
iii) $S = 100m$, v=? u=$10ms^{-1}$a=1

$v^2 = u^2 + 2as$
$v^2 = 10^2 + 2x1x100$
$v = 17.32m/s$

2) A particle moving in a straight line with a constant acceleration of $2ms^{-2}$ is initially at rest, find the distance covered by the particle in the 3rd second of its motion.

Solution

Using $S = ut + \frac{1}{2} at^2$
u=0m/s, t=2s and t=3s a= $2ms^{-2}$
t=2 : $s = 0x2 + ½\, x2x2^2 = 4m$
When t=3: a=$2ms^{-2}$ u=0m/s
$s = 0x3 + ½\, x2x3^2 = 9m$

Distance in 3rd
Distance for 3s – distance for 2s
$= 9 - 4 = 5m$
Distance in 3rds in 5m

3) A Travelling car A at a constant velocity of 25m/s overtake a stationery car B. 2s later car B sets off in pursuit , accelerating at a uniform rate of $6ms^{-2}$. How far does B travel before catching up with A

Solution

For A: $S_A = ut + \frac{1}{2} at^2$
Since it moves with a constant velocity a=0
S_A = 25t---------------(1)

For B: $S_B = ut + \frac{1}{2} at^2$

If B is to catch up with A then it must travel faster i.e it will take a time of (t-2)s
$S_B = 0x(t-2) + ½\, x6(t-2)^2$
$S_B = 3t^2 - 12t + 12$..............(2)

For B to catch A then
$S_A = S_B$
$25t = 3t^2 - 12t + 12$

$3t^2 - 37t + 12 = 0$
$t = \frac{37 \pm \sqrt{37^2 - 4x12x3}}{2x3}$
t=12s or $t = \frac{1}{3}s$
Since the car leaves 2s later then time 12s is correct since it gives a positive value
S_B = 25x12
S_B = 300m

4) A train travelling at $72kmh^{-1}$under goes uniform deceleration of $2ms^{-2}$,when brakes are applied. Find the time taken to come to rest and the distance travelled from the place where brakes are applied.

Solution

u= $\frac{72x1000}{60x60}$ =$20ms^{-1}$
a=$-2ms^{-2}$, v=0 comes to rest

$v = u + at$
$0 = 20 - -2xt$
$t = 10s$
$s = ut + ½\, at^2$

$s = 20x10 + 1/2\, x - 2x10^2$
s= 100m

EXERCISE:1

1. A particle is moving in a straight line with a constant acceleration of $6.0ms^{-2}$. As it pass a point A its sped is $20ms^{-1}$. What is its sped 10s after passing A **An[$80ms^{-1}$]**
2. A particle which is moving in a straight line with a velocity of $15ms^{-1}$ accelerates uniformly for 3.0s, increasing its velocity to $45ms^{-1}$. What distance does it travel while accelerating **An[90m]**
3. A car starts to accelerate at a constant rate of $0.80ms^{-2}$. It covers 400m while accelerating in the next 20s. what was the speed of the car when it started to accelerate **An[$12ms^{-1}$]**
4. A car moving at $30ms^{-1}$ is brought to rest with a constant retardation of $3.6ms^{-2}$. How far does it travel while coming to rest **An[125m]**

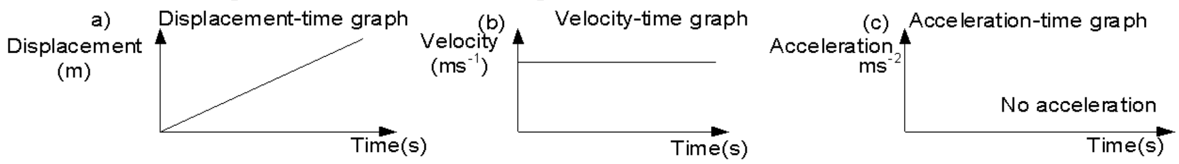
5. A car moving with a velocity of 54km/hr accelerates uniformly at a rate of 2ms^{-2}. Calculate the distance travelled from the place where acceleration began, given that final velocity reached is 72km/hr and find the time taken to cover this distance. **An [$43\frac{3}{4}m$, 2.5s]**

6. A bus travelling steadily at 30m/s along a straight road passes a stationary crab which, 5s later, begins to move with a uniform acceleration of 2ms^{-2} in the same direction as the bus
(a) How long does it take the car to acquire the same speed as the bus
(b) How far has the car travelled when it is level with the bus **An[15s, 1181m]**

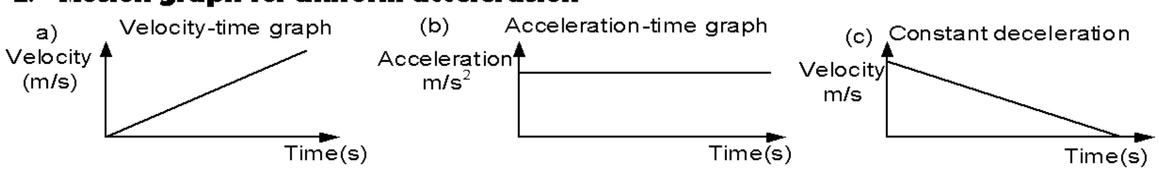
7. A body accelerates uniformly from rest at the rate of 6ms^{-2} for 15 seconds. Calculate
i) velocity reached within 15 seconds
ii) the distance covered within 15 seconds **An[90m/s, 675m]**

8. A particle moving on a straight line with constant acceleration has a velocity of 5ms^{-1} at one instant and 4s later it has a velocity of 15ms^{-1}. Find the acceleration and distance covered by particle.
An [a = 2.5ms^{-2}, s=40m]

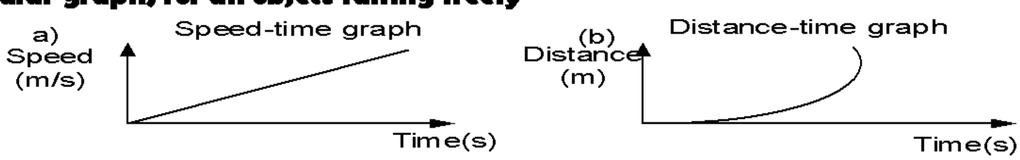
1. Motion graphs for uniform velocity



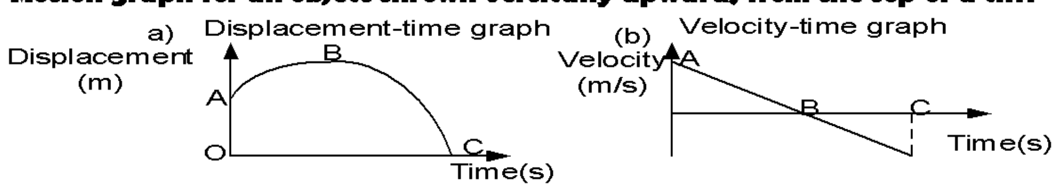
2. **Motion graph for uniform acceleration**



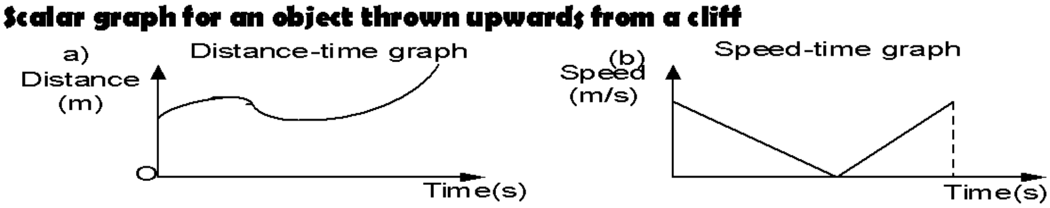
3. Scalar graphs for an object falling freely



4. Motion graph for an object thrown vertically upwards from the top of a cliff



5. Scalar graph for an object thrown upwards from a cliff



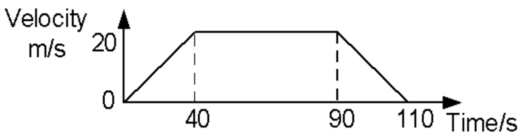
Note

For a body thrown vertically downwards,
$v = u + at$ becomes $v = u + gt$
$S = ut + \frac{1}{2}gt^2$ becomes $S = ut + \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$ becomes $v^2 = u^2 + 2gh$

For a body projected vertically upwards
$v = u + at$ becomes $v = u - gt$
$S = ut + \frac{1}{2}gt^2$ becomes $S = ut - \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$ becomes $v^2 = u^2 - 2gh$

Examples:

1. **A car started from rest and attained a velocity of *20m/s* in *40s*. It then maintained the velocity attained for *50s*. After that it was brought to rest by a constant breaking force in *20s*.**
   - **(i) Draw a velocity-time graph for the motion.**
   - **(ii) Using the graph, find the total distance travelled by the car.**
   - **(iii) What is the acceleration of the car?**



Total distance = Total area using each part

$$= \frac{1}{2}bh + LxW + \frac{1}{2}bh$$

$$= \left(\frac{1}{2} \text{ x } 40 \text{ x } 20\right) + (50 \, x \, 20) + \left(\frac{1}{2} \text{x } 20 \text{ x } 20\right)$$

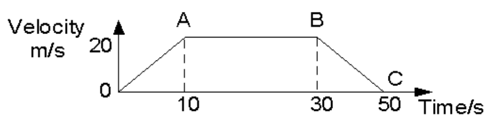
$$= 400 + 1000 + 200$$

$$= 1600m$$

2. **A car from rest accelerates steadily for 10s up to a velocity of 20m/s. It continues with a uniform velocity for a further 20s and then decelerates so that it stops in 20s**
   - **a) Draw a velocity-time graph to represent the motion**
   - **b) Calculate;**
     - **(i) Acceleration**
     - **(ii) Deceleration**
     - **(iii) Distance travelled**
     - **(iv) Average speed**

Solution

(a)



(i) Acceleration OA:

$$a = \frac{v-u}{t} = \frac{20-0}{10} = 2ms^{-2}$$

(ii) Deceleration BC:

$$a = \frac{v-u}{t} = \frac{0-20}{20} = -1 \, ms^{-2}$$

$$deceleration = 1 \, ms^{-2}$$

(iii) Distance =Area under graph

$$\left(\frac{1}{2}x10x20\right) + (20x20) + \left(\frac{1}{2}x20x20\right)$$

Distance = $700m$

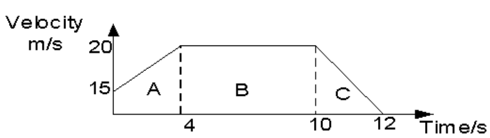
Method II(Area of a trapezium)

$$A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}x \, 20x(50+20) = 10 \, (70) = 700m$$

(iv) Average speed $= \frac{distance}{time} = \frac{700}{50} = 14m/s$

3. **The graph below shows the motion of the body**

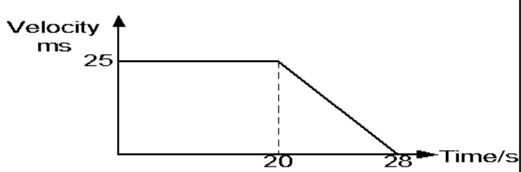


- **a) Describe the motion of the body**
- **b) Calculate the total distance travelled**

Solution

- **a) A body with initial velocity of 15m/s accelerates steadily to a velocity of 20m/s in 4s, it then continues with a uniform velocity for 6s and its brought to rest in 2s.**
- **b) Distance travelled=** $(4x15) + \left(\frac{1}{2}x4x5\right) + (20x6) + \left(\frac{1}{2}x20x2\right) = \ 210m$

4. **A car travelling at a speed of 90km/h for 20s and then brought to rest in 8s. Draw a velocity time graph and find the distance travelled.**



Distance travelled;

$$= (20x25) + \left(\frac{1}{2}x8x25\right)$$

$$= \ 600m$$

2.1.2: MOTION UNDER GRAVITY

1. Vertical motion

a) When a body is projected vertically upwards it experiences a uniform deceleration of $9.81ms^{-2}$. Its acceleration is given by $a = -g = 9.81ms^{-2}$. The equations of motion become

$v = u - gt$ | $S = ut - \frac{1}{2}gt^2$ | $v^2 = u^2 - 2gs$

b) An object freely falling vertically downwards has an acceleration of $a = g = 9.81ms^{-2}$.
The equations of motion become

$v = u + gt$ | $S = ut + \frac{1}{2}gt^2$ | $v^2 = u^2 + 2gs$

Definition

Acceleration due to gravity (g) is rate of change of velocity with time for an object falling freely under gravity.

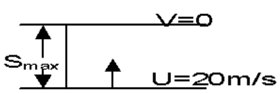
OR The force of attraction due to gravity exerted on a 1kg mass.

Free fall is motion resulting from a gravitational field that is not impeded by a medium that should provide a frictional retarding force or buoyancy.

Numerical examples

1. A ball is thrown vertically upwards with an initial speed of $20ms^{-1}$. Calculate.
   i) Time taken to return to the thrower ii) Maximum height reached

solution

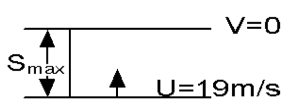


projected upwards; $v = u - gt$
At max height $v = 0$
$0 = 20 - 9.81t$
$t = 2.04s$
Time taken to reach maximum height $= 2.04s$

But the total time ta ken to return to the thrower $= 2t$
$= 2x2.04 = 4.08s$
$v^2 = u^2 - 2gs$
at max height v=0m/s, u=20m/s,
$g = 9.81ms^{-2}$
$0^2 = 20 - 2x9.81s_{max}$
$s_{max} = 20.39m$

2. A particle is projected vertically upwards with velocity of $19.6ms^{-1}$. Find
   i) The greatest height attained
   ii) Time taken by the particle to reach maximum height
   iii) Time of flight

Solution

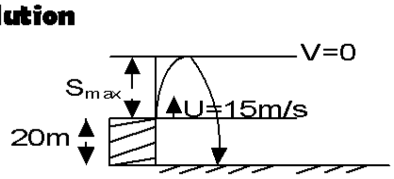


At greatest height $v = 0m/s$
$v^2 = u^2 - 2gs$
$0^2 = 19.6^2 - 2x9.81\ s_{max}$
$s_{max} = \frac{19.6^2}{2x9.81} = 19.58m$

ii) From $v = u - gt$
$u = 19.6, g = 9.81ms^{-2}$ $v = 0$ at max height
$0 = 19.6 - 9.81t$
$t = 1.998s$
Time to maximum height $= 2.0s$
iii) Time of flight $= 2x$ time to max height
$= 2x2 = 4.0s$

3. A man stands on the edge of a cliff and throws a stone vertically upwards at $15ms^{-1}$. After what time will the stone hit the ground 20m below the point of projection

Solution



v=0m/s at max height, s_{max}=? t=?
Method I: $v = u - gt$
$0 = 15 - 9.81t$
$t = 1.53s$

Time to maximum height = $1.53s$

$v^2 = u^2 + 2gs$

$0 = 15^2 - 2x9.81s_{max}$

$$s_{max} = \frac{15^2}{2x9.81} = 11.47m$$

Maximum height = $11.47m$

Total height = $(11.47 + 20) = 31.47m$

When the ball begins to return down from max height $u = 0m/s$

$$S = ut + \frac{1}{2}gt^2$$

$31.47 = 0xt + ½\, x9.81t^2$

$$t = \sqrt{\frac{31.47x2}{9.81}} = 2.53s$$

Total time = $(2.53 + 1.53) = 4.06s$

Time taken to hit the ground = $4.06s$

Method II

The height of the cliff = 20m which is below the point of project therefore

$s = -2m\; u = 15m/s$

$$S = ut - \frac{1}{2}gt^2$$

$-20 = 15t - ½\, x9.81t^2$

$-20 = 15t - 4.905t^2$

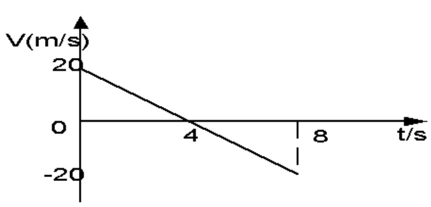
$t = 4.06s$

Time taken to hits the ground = $4.06s$

4. A car decelerates uniformly from 20ms^{-1} to rest in 4s, then reverses with uniform acceleration back to it original starting point also in 4s
   a) Sketch the velocity-time graph for the motion, and use it to determine the displacement and average velocity
   b) Sketch the speed-time graph for the motion and use it to determine the total distance covered and the average speed.

Solution

Velocity-time graph



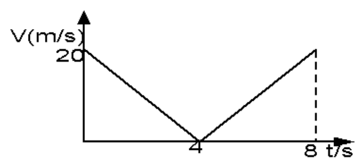
Displacement $s = ½\, bh + ½\, bh$

$= ½\, x4x20 + ½\, x4x(-20)$

$s = 40 - 4 = 0m$

Average velocity = $\frac{total\ displacement}{total\ time} = \frac{0}{8} = 0m/s$

Speed-time graph



Total distance = $½\, x20x4 + ½\, x20x4$

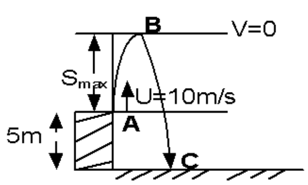
$= 80m$

Average speed = $\frac{total\ distance}{total\ time} = \frac{80}{8} = 10ms^{-1}$

UNEB 2003 No 1 b(ii)

A ball is thrown vertically upwards with a velocity of 10ms^{-1}from a point 50m above the ground. Describe with the aid of a velocity - time graph, the subsequent motion of the ball. (10marks)

Solution



Time to reach max height v=0,

$v = u - gt$

$0 = 10 - 9.81t$

$t = 1.02s$

Time to reach maximum height is 1.02s

At Max height $v = 0$

$v^2 = u^2 - 2gs$

$0 = 10^2 - 2x9.81Smax$

$S_{max} = 5.1m$

Total height = $(5.1 + 5)$

$= 10.1m$

Time taken to move from max height to the ground is

t=?, u=0m/s g=9.81ms^{-2}

$$S = ut + \frac{1}{2}gt^2$$

$10.1 = 0xt + \frac{1}{2}x9.81t^2$

$$t = \sqrt{\frac{20.2}{9.81}} = 1.43s$$

Final velocity when the ball hits the ground $v = ?$

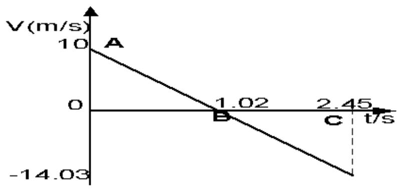
$u = 0, t = 1.43s, g = 9.81ms^{-1}$

$v = ut + gt$

$v = 0 + 9.81x1.43$

$= 14.03m/s$

Velocity-time graph



✓When the ball is thrown vertically upwards with a velocity of $10ms^{-1}$ it decelerates uniformly at $9.81ms^{-1}$ til its velocity reaches zero at B(maximum height).

✓The time taken to reach maximum height B is 1.02s and the maximum height is 5.1m

✓After reaching the maximum height, the ball begins to fall downwards with a uniform acceleration of $9.81m/s^2$ but the direction is now opposite and therefore the velocity is negative until it reaches a final velocity of 14.03m/s in a time of 2.45s from the time of projection.

Exercise :2

1. A pebble is dropped from rest at the top of a cliff 125m high.
   (a) How long does it take to reach the foot of the cliff and with what speed does it hit the floor
   (b) With what speed must a second pebble be thrown vertically down wards from the cliff top if it is to reach the bottom in 4s . **An(5s, 50m/s, 11.25m/s)**
2. A stone is thrown horizontally from the top of a vertical cliff with velocity 15m/s is observed to strike the horizontal ground at a distance of 45m form the base of the cliff. What is;
   (a) The height of the cliff. **An(45m, 63.4°)**
   (b) The angle the path of the stone makes with the ground at the moment of impact
3. A ball is thrown vertically upwards and caught by the thrower on its return. Sketch a graph of velocity against time, neglecting air resistance
4. A ball is dropped from a cliff top and takes 3s to reach the beach below. Calculate
   a) The height of the cliff **An(44.1m)**
   b) Velocity acquired by the ball **An(29.4m/s)**
5. With what velocity must a ball be thrown upwards to reach a height of 15m **An(17.1ms^{-1})**
6. A stone is dropped from the top of a cliff which is 80m high. How long does it take to reach the bottom of the cliff **An(4.0s)**
7. A stone is fired vertically upwards from a catapult and lands 5.0s later.
   a) What was the initial velocity of the stone
   b) For how long was the stone at a height of 20m or more **An(25ms^{-1}, 3.0s)**
8. A stone is thrown vertically upwards at $10ms^{-1}$ from a bridge which is 15m above a river
   (a) What is the speed of the stone as it hits the river
   (b) With what speed would it hit the river if it were thrown downwards at $10ms^{-1}$ **An(20ms^{-1}, 20ms^{-1})**

UNEB 2014 No 1(c)

(i) State **Newton's laws of motion** (03marks)

(ii) Explain how a rocket is kept in motion (04marks)

(iii) Explain why passengers in a bus are thrown backwards when the bus suddenly starts moving. (03marks)

UNEB 2013 No 3(d)

(i) Define uniformly accelerated motion (03marks)

(ii) A train starts from rest at station **A** and accelerates at 1.25 ms^{-2} until it reaches a speed of 20 ms^{-1}. It then travels at this steady speed for a distance of 1.56km and then decelerates at 2 ms^{-2} to come to rest at station **B**. Find the distance from **A** and **B**

An (1820m) (04marks)

UNEB 2011 No 1(a)

Define the following terms

(i) Uniform acceleration (01mark)

(ii) Angular velocity (01 mark)

UNEB 2010 No 1(d)

(i) Define uniform acceleration (01 mark)

(ii) With the aid of a vel-time graph, describe the motion of a body projected vertically upwards (03 marks)

UNEB 2009 No 2

a) Define the following terms

(i) Velocity

(ii) Moment of a force (02marks)

b) i) A ball is projected vertically upwards with a speed of $50ms^{-1}$, on return it passes the point of projection and falls 78m below. Calculate the total time taken **An(11.57s)** (05 marks)

UNEB 2008 No 1(a)

i) Define the terms velocity and displacement (02 marks)

ii) Sketch a graph of velocity against time for an object thrown vertically upwards (02 marks)

UNEB 2007 No 4(b)(i) What is meant by acceleration due to gravity

UNEB 2006 No 1

a) i) What is meant by uniformly accelerated motion (01 mark)

ii) Sketch the speed against time graph for a uniformly accelerated body (01 mark)

b) (i) Derive the expression $S = ut + \frac{1}{2}at^2$

For the distance S moved by a body which is initially travelling with speed u and is uniformly accelerated for time t (04 marks)

UNEB 1993 No 1

(a) Define the terms

(i) Displacement

(ii) Uniform acceleration

(b) i) A stone thrown vertically upwards from the top of a building with an initial velocity of 10m/s. the stone takes 2.5s to land on the ground.

ii) Calculate the height of the building

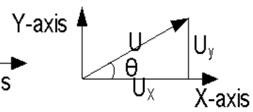
(iii)State the energy changes that occurred during the motion of the stone (03 marks)

2. PROJECTILE MOTION

This is the motion of a body which after being given an initial velocity moves under the influence

Consider a ball projected at O with an initial velocity um/s at an angle θ to the horizontal.

Resolving initial velocity



From the figure: $\sin\theta = \frac{u_y}{u}$

$u_y = u\sin\theta$ ------------------(1)

Also: $\cos\theta = \frac{u_x}{u}$

$u_x = u\cos\theta$ ------------------(2)

Equation (1) is the initial vertical component of velocity

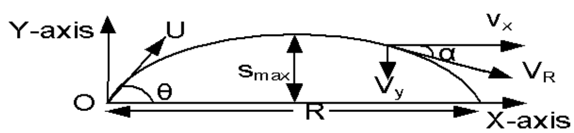
Equation (2) is the initial horizontal component of velocity

Note

The horizontal component of velocity $[u_x = u\cos\theta]$ is constant through the motion and therefore the acceleration is zero.

MATHEMATICAL FORMULAR IN PROJECTILES

All formulas in projectiles are derived from equations of linear motion



Finding velocity at any time t.

Horizontally: $v = u_x + at$

$u_x = u\cos\theta$,

$a = 0$ (constant velocity)

$$v_x = u\cos\theta$$

Vertically: $v = u_y + at$

$u_y = u\sin\theta$

a= −g

$$v_y = u\sin\theta - gt$$

Velocity at any time t

$$v = \sqrt{V_x^2 + V_y^2}$$

Direction of motion

$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$ to the horizontal

Finding distances at any time t

horizontally : $s_x = u_x t + ½ at^2$

$u_x = u\cos\theta$, a= 0

$$x = u\cos\theta\, t$$

Vertically: $s_y = u_y t + ½ at^2$

$u_y = u\sin\theta$, $a = g$

$$y = u\sin\theta t - \frac{1}{2}gt^2$$

TERMS USED IN PROJECTILES

1. MAXIMUM HEIGHT [GREATEST HEIGHT] [S_{max}]

For vertical motion : at max height v=0,

$u_y = u\sin\theta$, $a = -g$, s= S_{max}

$v_y{}^2 = u_y{}^2 + 2gs$

$0 = (u\sin\theta)^2 - 2gS_{max}$

$2gS_{max} = u^2\sin^2\theta$

$$Smax = \frac{u^2\sin^2\theta}{2g}$$

Note : $\sin^2\theta = (\sin\theta)^2$ but $\sin^2\theta \neq \sin\theta^2$

2. TIME TO REACH MAX HEIGHT [t]

Vertically $v = u_y + at$ at max height v=0

$u_y = u\sin\theta$, $a = g$

$0 = u\sin\theta - gt$

$$t = \frac{u\sin\theta}{g}$$

3. TIME OF FLIGHT [T]

It refers to the total time taken by the projectile to move from the point of projection to the point where it lands on the horizontal plane through the point of projection.

Vertically: $S_y = u_y t + ½ at^2$

at point A when the projectile return to the plane S_y=0,

t=T(time of flight), $a = -g$ $u_y = u \sin\theta$

$$0 = u\sin\theta T - \frac{gT^2}{2}$$

$$T\left(u\sin\theta - \frac{gT}{2}\right) = 0$$

Either $T = 0$ or $\left(u\sin\theta - \frac{gT}{2}\right) = 0$

$$\left(u\sin\theta - \frac{gT}{2}\right) = 0$$

$$u\sin\theta = \frac{gT}{2}$$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

Note: The time of flight is twice the time to maximum height

4. RANGE [R]

It refers to the horizontal distance from the point of projection to where the projectile lands along the horizontal plane through the point of projection.

Neglecting air resistance the horizontal component of velocity $u\cos\theta$ remains constant during the flight

Horizontally : $S_x = u_x t + ½ at^2$

$u_x = u\cos\theta$, a=0 (constant velocity), t=T

$$R = u\cos\theta\, T + ½ x 0 x T^2$$

$$R = u\cos\theta T$$

But $T = \frac{2u\sin\theta}{g}$

$$R = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

But from trigonometry $2\sin\theta\cos\theta = \sin 2\theta$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

5. MAXIMUM RANGE [R_{max}]

For maximum range sin2θ =1, R=R_{max}

2 θ= $\sin^{-1}(1)$

$$2\theta = 90°$$

$$R_{max} = \frac{u^2 \sin 90}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

6. EQUATION OF A TRAJECTORY

A trajectory is a path described by a projectile.

A trajectory is expressed in terms of horizontal distance x and vertical distance y.

For horizontal motion at any time t

$$x = u\cos\theta t$$

$t = \frac{x}{u\cos\theta}$ ----------------[1]

For vertical motion at any time t

$y = u\sin\theta t - ½ gt^2$ -------------[2]

Putting t into equation [2]

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

since $y = ax - bx^2$

the motion is parabolic

Either $\boxed{y = x\tan\theta - \frac{g x^2 \sec^2\theta}{2u^2}}$

Or $\boxed{y = x\tan\theta - \frac{g x^2 (1+\tan^2\theta)}{2u^2}}$

A. Objects projected upwards from the ground at an angle to the horizontal

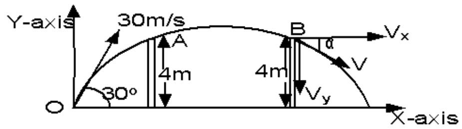
1. A Particle is projected with a velocity of $30ms^{-1}$ at an angle of elevation of 30°. Find
   i) The greatest height reached
   ii) The time of flight
   iii) Horizontal range
   iv) The velocity and direction of motion at a height of 4m on its way downwards

Solution

(i) $Smax = \frac{u^2 \sin^2\theta}{2g} = \frac{30^2 \sin^2 30}{2x9.81} = 11.47m$

(ii) $T = \frac{2u\sin\theta}{g} = \frac{2 x 30 \sin 30}{9.81} = 3.06s$

(iii) R= $\frac{u^2 \sin 2\theta}{g} = \frac{30^2 \sin 2x30}{9.81} = 79.45m$



For vertical motion

$y = usin\theta t - ½ gt^2$

$4 = 30sin30t$ – ½ $9.81x\, t^2$

$4.905t^2 - 15t + 4 = 0$

$t = 2.76s$ or $t = 0.30s$

The value of $t = 0.30s$ is the correct time since it's the smaller value for which the body moves upwards.

$v_x = ucos\theta$

$v_x = 30cos\, 30 = 25.98m/s$

$v_y = usin\theta - gt$

$v_y = 305x30 - 9.81x0.30 = 12.06m/s$

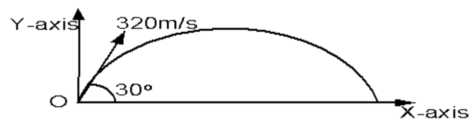
$v = \sqrt{V_x^2 + V_y^2} = \sqrt{25.98^2 + 12.06^2} = 28.64m/s$

Direction : $\alpha = tan^{-1}\frac{V_y}{V_x} = tan^{-1}\left(\frac{12.06}{25.98}\right)$ =**24.9°**

Velocity is 28.64m/s at 24.9° to horizontal

2. **A projectile is fired with a velocity of 320m/s at an angle of 30° to the horizontal. Find**
   (i) time to reach the greatest height
   (ii) its horizontal range
   (iii) maximum range

Solution



i) At max height $v = 0$,

$v = usin\theta - gt$

$0 = 320sin30 - 9.81t$

$t = \frac{320sin30}{9.81} = 16.31s$

ii) range $R = ucos\theta x$time of flight

Time of flight =twice time to max height

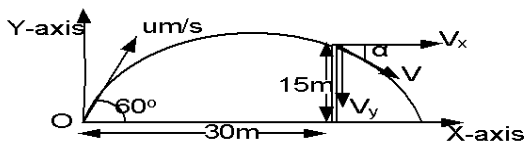
R= $320cos30x\,[2x16.31] = 9039.92m$

iii) max range

$R_{max} = \frac{u^2}{g} = \frac{320^2}{9.81} = 10438.33\text{m}$

3. **A projectile fired at an angle of 60° above the horizontal strikes a building 30m away at a point 15m above the point of projection. Find**
   (i) Speed of projection
   (ii) Velocity when it strikes a building

Solution



(i) Horizontal distance at time t : $x = ucos\theta t$

$30 = utcos60$

$t = \frac{60}{u}$

Also vertical distance at any time t

$y = usin\theta - \frac{1}{2}gt^2$

$15 = usin60x\frac{60}{u} - \frac{1}{2}x9.81\left(\frac{60}{u}\right)^2$

$15 = 51.96152423 - \frac{4.905\,x\,3600}{u^2}$

$u = \sqrt{477.7400383} = 21.86m/s$

ii)but since $t = \frac{60}{u}$

$t = \frac{60}{21.86} = 2.75s$

$v_x = ucos\theta$

$v_x = 21.86cos60 = 10.93ms^{-1}$

$v_y = usin\theta - gt$

$v_y = 21.81sin60 - 9.81x2.75 = -8.09ms^{-1}$

velocity at any time

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{10.93^2 + (-8.09)^2}$

$= 13.60ms^{-1}$

$\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right) = tan^{-1}\left(\frac{8.09}{10.9}\right) = 36.6°$

The velocity is $13.60ms^{-1}$at $36.6°$to the horizontal

Alternatively

$y = xtan\theta - \frac{gx^2}{2u^2COS^2\theta}$

$y = 15m, x = 30m, \theta = 60°, u = ?$

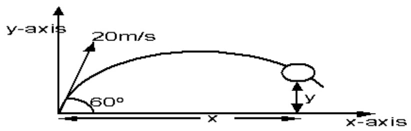
$15 = 30tan60 - \frac{9.81x30^2}{2u^2Cos^260}$

$15 = 51.96152423 - \frac{17658}{u^2}$

$u = \sqrt{477.7400383} = 21.86m/s$

4. A body is projected at an angle of 60° above horizontal and passes through a net after 10s. Find the horizontal and vertical distance moved by the body after it, was projected at a speed of 20m/s

Solution



Horizontal motion : $x = ucos\theta xt$

$x = 20cos60x10$

$x = 100m$

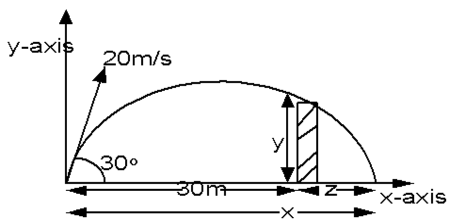
Vertical motion; $y = usin\theta t - ½\ gt^2$

$y = 20(sin60)x10 - ½\ x9.81x10^2$

$y = -317.29m$

5. A ball is kicked from the spot 30m from the goal post with a velocity of 20m/s at 30° to the horizontal. The ball just clears the horizontal bar of a goal post. Find;
   (i) Height of the goal post
   (ii) How far behind the goal post does the ball land

Solution



horizontal motion : $x = ucos\theta xt$

$30 = 20cos30xt$

$t = 1.732s$

For vertical motion: $y = usin\theta t - ½\ gt^2$

$y = (20sin30)x1.732 - \frac{1}{2}x9.81x(1.732)^2$

$y = 2.61m$

Height of the goal post = 2.61m

ii) Time of flight

$T = \frac{2\ u\ \sin\theta}{g} = \frac{2x20xsin30}{9.81} = 2.04s$

iii) Horizontal distance: $x = ucos\theta xt$

$x = 20cos30x2.04$ =**35.33m**

but $x = 20 + z$

$35.33 = 30 + z$

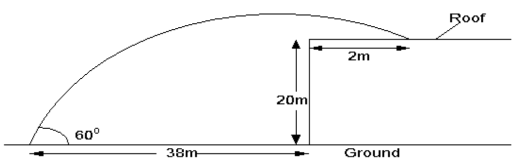
$z = 5.33m$ The ball 5.33m behind the goal

EXERCISE : 3

1. A particle is projected at an angle of 60° to the horizontal with a velocity of 20m/s. calculate the greatest height the particle attains **An[15.29m]**
2. A stone is projected at an angle of 60° to the horizontal with a velocity of 30m/s. calculate;
   (a) the highest point reached
   (b) Range
   (c) Time taken for flight
   (d) Height of the stone at the instant that the path makes an angle of 30° with the horizontal
   An[33.75m, 78m, 5.2s, 33.3m]
3. A particle is projected from level ground towards a vertical pole, 4m high and 30m away from the point of projection. It just passes the pole in one second. Find
   i) Its initial speed and angle of projection **An [39.29m/s, 16.5°]**
   ii) The distance beyond the pole where the particle will fall **An [24.42m]**
4. A particle is projected with a velocity of 30m/s at an angle of 40° above the horizontal plane. find ;
   a) The time for which the particle is in the air.
   b) The horizontal distance it travels **An [3.9s, 22.9m/s]**
5. A body is projected with a velocity of 200ms^{-1} at an angle of 30° above the horizontal. Calculate
   (a) Time taken to reach the maximum height
   (b) Its velocity after 16s **An [10.2s, 183m/s at 19.1°]**
6. A particle is projected from a level ground in such a way that its horizontal and vertical components of velocity are 20ms^{-1} and 10ms^{-1} respectively. Find
   (a) Maximum height of the particle
   (b) Its horizontal distance from the point of projection when it returns to the ground
   (c) The magnitude and direction of the velocity on landing **An [5.0m, 40m, 22.4m/s at 26.6° below horizontal]**
7. A particle is projected with a speed of 25ms^{-1} at 30° above the horizontal. Find;

(a) Time taken to reach the height point of trajectory
(b) The magnitude and direction of the velocity after 2.0s **An [1.25s, 22.9m/s at 19.1° below horizontal]**

8. A projectile is launched with a velocity of 1800m/s at an angle 60° with the horizontal. Determine the speed of the projectile at a height of 32km when falling downwards **An[1616.23m/s]**
9. A hammer thrown in athletics consists of a metal sphere of mass 7.26kg with a wire handle attached, the mass of which can be neglected. In a certain attempt it is thrown with an initial velocity which makes an angle of 45° with the horizontal and its flight takes 4.00s. stating any assumptions find;
   (i) The horizontal distance travelled
   (ii) Kinetic energy of the sphere just before it strikes the ground **An [80.0m, $2.90x10^3$J]**
10. A soft ball is thrown at an angle of 60 above the horizontal. It lands a distance 2m from the edge of a flat roof of height 20m. the edge of the roof is 38m horizontally from the thrower.

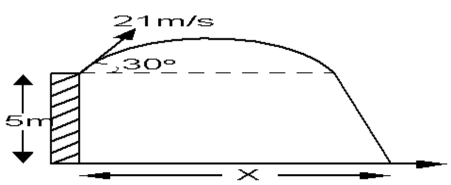


Calculate
   (i) The speed at which the ball was thrown **An (25.4 ms^{-1})**
   (ii) The velocity with which the ball strikes the roof **An (15.64 ms^{-1} $at\ 36.2°$ below the horizontal)**
11. A stone thrown upwards at an angle θ to the horizontal with speed ums^{-1} just clears a vertical wall 4m high and 10m from the point of projection when travelling horizontally. Find the angle of projection **An[38.66°]**

B. Objects projected upwards from a point above the ground at an angle to the horizontal

1. A particle is projected at an angle of elevation of 30° with a speed of 21m/s. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before striking the ground

Solution



$u = -5m$ since it's below the point of projection

For vertical motion: $y = usin\theta t - ½ gt^2$

$$-5 = 21sin30t - \frac{9.81t^2}{2}$$

$4.905t^2 - 10.5t - 5 = 0$

t= $2.54s$ or t = $-0.40s$

Time of flight $t = 2.54s$

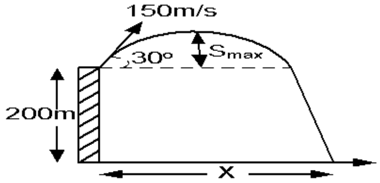
For horizontal motion

$x = ucos\theta xt = 21(cos30)\,x2.54 = 46.19m$

The horizontal distance = $46.19m$

2. A bullet is fired from a gun placed at a height of 200m with a velocity of $150ms^{-1}$ at an angle of 30° to the horizontal find
   i) Maximum height attained
   ii) Time taken for the bullet to hit the ground

Solution



i) $Smax = \frac{u^2\,sin^2\theta}{2g} = \frac{150^2\,sin^2 30}{2x9.81} = 286.70m$

The max height attained is 286.70m from the point of projection

ii) Time taken for the bullet to hit the ground

Vertical motion : $y = usin\theta t - ½\,gt^2$

$y = -200m$ since its below the point of projection

$-200 = 150sin30t - ½\,x9.81t^2$

$-200 = 75t - 4.905t^2$

$t = 17.61s\ or\ t = -2.32s$

Time taken is 17.61s

Trial :1

1. A particle is projected with a velocity of $10ms^{-1}$ at an angle of 45^0 to the horizontal, it hits the ground at a point which is 3m below its point of projection. Find the time fro which it is in the air and the horizontal distance covered by the particle in this time **An[1.76s, 12.42m]**
2. A pebble is thrown from the top of a cliff at a speed of 10m/s and at 30^0 above the horizontal. it hits the sea below the cliff 6.0s later , find;
   a) The height of the cliff . **An[150m, 52m]**
   b) The distance from the base of the cliff at which the pebble falls into the sea.

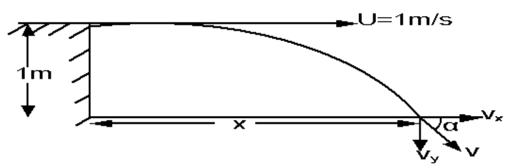
C. An object projected horizontally from a height above the ground

Examples

1. A ball rolls off the edge of a table top 1m high above the floor with a horizontal velocity $1ms^{-1}$. Find;

i) The time it takes to hit the floor
ii) The horizontal distance it covered
iii) The velocity when it hits the floor

Solution



$u=1ms^{-1}$ $\theta=0^0$ y=-1m below the point of projection

vertical motion: $y = usin\theta t - ½ gt^2$

$-1 = 1xsin0xt - ½ x9.81t^2$

$-1 = -4.905t^2$

$t = 0.45s$

ii) $x = ucos\theta t = 1xcos0x0.45 = 0.45m$

iii) velocity when it hits the ground

$v_x = ucos\theta = 1cos0 = 1m/s$

$v_y = usin\theta - gt$

$v_y = 1sin0 - 9.81x0.45 = -4.4m/s$

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{(1)^2 + (-4.4)^2} = 4.5ms^{-1}$

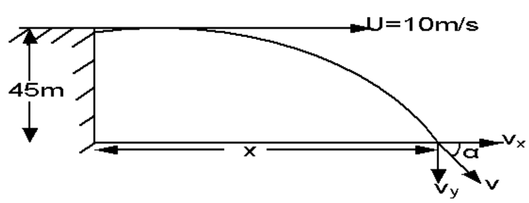
Direction: $\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right) = tan^{-1}\left(\frac{4.4}{1}\right) = 77.2°$

The velocity is $4.5ms^{-1}$ at $77.2°$to the horizontal

2. A ball is thrown forward horizontally from the top of a cliff with a velocity of 10m/s. the height of a cliff above the ground is 45m. calculate

i) Time to reach the ground
ii) Distance from the cliff where the ball hits the ground
iii) Direction of the ball just before it hits the ground

Solution



$u=10ms^{-1}$ $\theta=0^0$ y=-45m below the point of projection

For vertical motion

$y = usin\theta t - ½ gt^2$

$-45 = 10xsin0xt - ½ x9.81t^2$

$t = 3.03s$

ii) $x = ucos\theta t = 10xcos0x3.03 = 30.3m$

iv) velocity when it hits the ground

$v_x = ucos\theta = 10cos0 = 10m/s$

$v_y = usin\theta - gt$

$v_y = 10sin0 - 9.81x3.03 = 29.72m/s$

$v = \sqrt{V_x^2 + V_y^2} = \sqrt{(10)^2 + (29.72)^2} = 31.36ms^{-1}$

$\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right) = tan^{-1}\left(\frac{29.72}{10}\right) = 71.4°$

The velocity is $31.36ms^{-1}$ at $71.4°$to the horizontal

Trial:2

1. A pencil is accidentally knocked off the edge of a horizontal desktop. The height of the desk is 64.8cm and the pencil hits the floor a horizontal distance of 32.4cm from the edge of the desk, What was the speed of the pencil as it left the desk. **An[$0.9ms^{-1}$]**

2. An aero plane moving horizontally at 150ms^{-1} releases a bomb at a height of 500m. the hits the intended target. What was the horizontal distance of aero plane from the target when the bomb was released. **An(1500m)**

UNEB 2016 No1 (b)

A particle is projected from a point on a horizontal plane with a velocity, u, at an angle, θ, above the horizontal. Shwo that the maxmum horizontal range R_{max} is given by $R_{max} = \frac{u^2}{g}$ where g is acceleration due to gravity. (04marks)

UNEB 2014 No1 (a)

(i) What is a **projectile motion** (01marks)

(ii) A bomb is dropped from an aero plane when it is directly above a target at a height of 1402.5m. the aero plane is moving horizontally with a speed of 500kmh^{-1}. Determine whether the bomb will hit the target. **An (misses target by 2347.2m)** (05marks)

UNEB 2012 No 3 (d)

(i) Derive an expression for maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projection θ to the horizontal [02 marks]

(ii) Sketch a graph to show the relationship between kinetic energy and height above the ground in a projectile.

UNEB 2010 No (d)

iii) Calculate the range of a projectile which is fired at an angle of 45° to the horizontal with a speed of 20m/s **An [40.77m]**

UNEB 2009 No 1 (d)

A stone is projected at an angle of 20° to the horizontal and just clears a wall which is 10m high and 30m form the point of projection. Find the;

i) Speed of projection (04marks)

ii) Angle which the stone makes with the horizontal as it clears the wall (03marks)

An[73.78m/s, 16.9°]

UNEB 2006 No 1 (c)

A projectile is fired horizontally from the top of a cliff 250m high. The projectile lands 1.414x10^{3}m from the bottom of the cliff. Find the

i) Initial speed of the projectile (05 marks)

ii) Velocity of the projectile just before it hits the ground (05 marks)

An [198m/s, 210m/s at 19.5°]

UNEB 2000 No 3 (b)

(i) Define the terms time of flight and range as applied to projectile motion (02 marks)

(ii) A projectile is fired in air with a speed um/s at an angle θ to the horizontal. Find the time of flight of the projectile (02marks)

MARCH UNEB 1995 No 1

a) (i) write the equation of uniformly accelerated motion (03 marks)

(ii) Derive the expression for the maximum horizontal distance travelled by a projectile in terms of the initial speed u and the angle of projectile θ to horizontal (04 marks)

b) A bullet is fired from a gun placed a height of 200m with a velocity of 150m/s at an angle of 30° to the horizontal. Find

i) The maximum height attained

ii) The time for the bullet to hit the ground (07marks)

CHAPTER 3: COMPOSITION AND RESOLUTION OF VECTORS

3.1.0: VECTOR QUANTITY

It is a physical quantity with both magnitude and direction.
Example; displacement, velocity, acceleration, force, weight and momentum

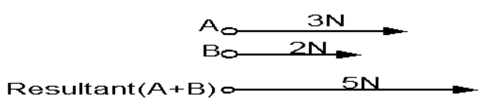
3.1.2: SCALAR QUANTITY

It is a physical quantity with only magnitude.
Example; distance, speed, time, temperature, mass and energy

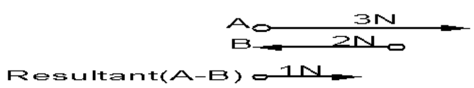
3.1.3: VECTOR ADDITION

A. Vectors acting in the same line

i) If vectors are acting in the same direction then resultant along that direction is just the sum of the two vectors.



ii) If they are moving in the opposite direction then, the resultant is difference of the vectors but along the direction of the bigger vector.



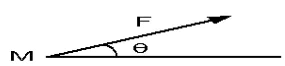
B . vectors acting at an angle

With vectors inclined at an angle to each other, a triangle of vectors is used to find the resultant. The resultant given by the line that completes the triangle.

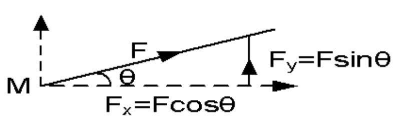
Components of a vector

The component of a vector is the effective value of a vector a long a particular direction. The component along any direction is the magnitude of a vector multiplied by the **cosine of the angle** between its direction and the direction of the component.

Suppose a force F pulls a body of mass m along a truck at an angle θ to the horizontal as shown below;



The effective force that makes the body move along the horizontal is the component of F along the horizontal



$Cos\theta = \frac{F_x}{F}$

$$\boxed{F_x = Fcos\theta}$$

$Sin\theta = \frac{F_y}{F}$

$$\boxed{F_y = Fsin\theta}$$

$$\boxed{\text{Resultant vector } \boldsymbol{F_R} = \sqrt{\boldsymbol{F_x^2} + \boldsymbol{F_y^2}}}$$

$$\boxed{\text{Direction}\alpha = \boldsymbol{tan^{-1}}\left(\frac{\boldsymbol{F_y}}{\boldsymbol{F_x}}\right)}$$

Hints

When a vector is inclined at an angle θ to the horizontal then;

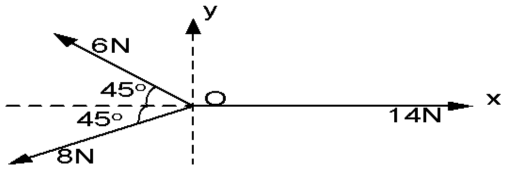
- Along the horizontal, the component of the vector is $cos\theta$
- Along the vertical, the component of the vector is $sin\ \theta$

When a vector is inclined at θ to the vertical then;

- Along the horizontal, the component of the vector is $sin\ \theta$
- Along the vertical, the component of the vector is $cos\theta$

Examples

1. Three forces are applied to a point as shown below



Calculate

a) The component in directions Ox and Oy respectively

b) Resultant force acting at O

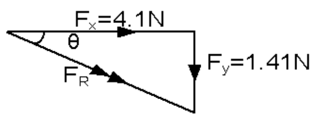
Solution

Components along Ox

$F_x = 14 - 6cos45 - 8cos45 = 4.10N$

Component along Oy

$F_y = 6sin45 - 8sin45 = -1.41N$



$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{4.1^2 + (-1.41)^2} = 4.34N$

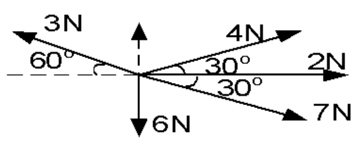
Direction $\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{1.41}{4.1}\right) = 19.0°$

Resultant force is $4.34N$ at $19.0°$ below the horizontal

2. Forces of 2N, 4N, 3N, 6N, and 7N act on a particle in the direction 0°, 30°, 120°, 270° and 330° respectively. Find the magnitude and direction of a single force represented by the above forces.

Solution

Note: the directions given involve 1,2 and 3 digits there fore they are angles and must be read anticlockwise starting from the positive x-axis

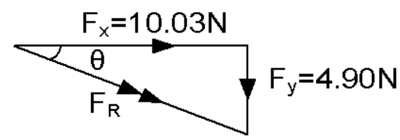


Resultant component along x-axis

$F_x = 2 + 4cos30 + 7cos30 - 3cos60 = 10.03N$

Resultant component along y-axis

$F_y = 4sin30 + 3sin60 - 7sin30 - 6 = -4.90N$



$F_R = \sqrt{10.03^2 + (-4.90)^2} = 11.16N$

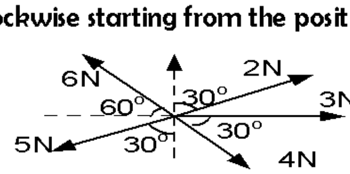
$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{4.90}{10.03}\right) = 26.04°$

The resultant force is 11.16N at 26.04° below the horizontal.

3. Forces of 2N, 3N, 4N, 5N, and 6N act on a particle in the direction 030°, 090°, 120°, 210°, and 330° respectively. Find the resultant force.

Solution

Note: the directions given involve 3 digits there fore they are bearings and must be read clockwise starting from the positive y-axis



Resultant along the x-axis

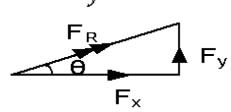
$F_x = 3 + 2sin30 + 4cos30 - 5cos30 - 6cos60$

$F_x = 1.964N$

Resultant along the y-axis

$F_y = 6sin60 + 2cos30 - 5cos30 - 4sin30$

$F_y = 0.598N$



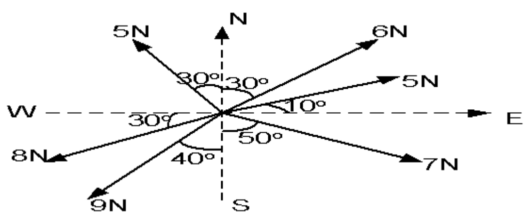
$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{1.964^2 + 0.598^2} = 2.053N$

Direction $\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = 16.9°$

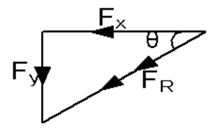
The resultant force is 2.053N at 16.9° above the horizontal

4. Forces of 6N, 5N, 7N, 8N, 5N, and 9N act pm a particle in the direction N30°E, N30°W, S50°E, N60°W, N80°E and S40°W, respectively. find the resultant force.

Solution



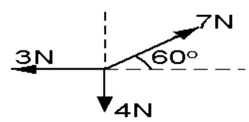
$$F_x = 5cos10 + 6sin30 + 7sin50 - 9sin40 - 8cos50 - 5sin30 = -1.927N$$
$$F_y = 5cos30 + 6cos30 + 5sin10 - 8sin30 - 9cos40 - 7cos50 = -4.999N$$



$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{1.927^2 + 4.999^2} = 5.36N$$
$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{4.999}{1.927}\right) = 68.9°$$

Resultant force is 5.36N at 68.9° below horizontal

5. **A particle at the origin O is acted upon by the three forces as shown below. Find the position of the particle after 2 seconds of its mass is 1kg.**



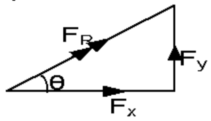
Solution

Resultant along horizontal

$$F_x = -3 + 7cos60 = 0.5N$$

Resultant along vertical

$$F_y = 7sin60 - 4 = 2.06N$$



$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{0.5^2 + 2.06^2} = 2.12N$$

But $F_R = ma$

$2.12 = 1a$

$a = 2.12ms^{-2}$

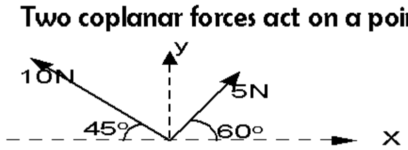
From $S = ut + ½\, at^2$

$u = 0 \;\; t = 2s \;\; a = 2.12ms^{-2}$

$S = 0x2 + ½\, x2.12x2^2 = 4.24m$

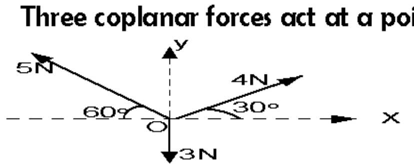
EXERCISE:4

1. A force of 3N acts at 60° to a force of 5N. find the magnitude and direction of their resultant
   An(7N at 21.8° to the 5N force)
2. A force of 3N act at 90° to a force of 4N. Find the magnitude and direction of their resultant
   An(5N at 37° to the 4N force)
3. Two coplanar forces act on a point O as shown below



Calculate the resultant force
An[12.3N at 68.0° above the horizontal]

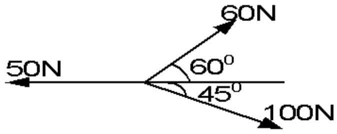
4. Three coplanar forces act at a point as shown below



Find the resultant force acting at O
An[3.4N at 73.1° above the horizontal]

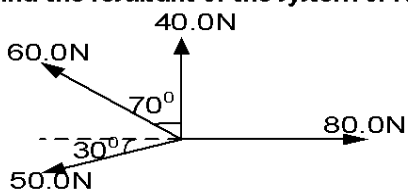
5. Forces of 2N, 1N, 3N and 4N act on a particle in the directions 0°, 90°, 270° and 330° respectively. Find the magnitude and direction of the resultant force.
   An[6.77N at 36.2° below the horizontal]

6. Forces of 7N, 2N, 2N, and 5N act on a particle in the direction 060°, 160°, 200 ° and 315° respectively. Find the resultant force. **An[4.14N at 52.36° below the horizontal]**
7. Calculate the magnitude and direction of the resultant of the forces shown below



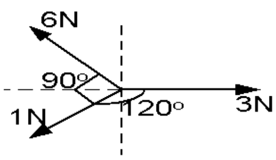
An(54.1N at 20° below the horizontal)

8. Find the resultant of the system of forces



An(40.6N at 61.0° to horizontal)

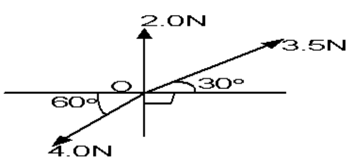
9. Three forces act on a body of mass 0.5kg as shown is the diagram. Find the position of the particle after 4 seconds.



An[3.44N, 6.88ms^{-2}, 55.2m]

UNEB 2008 No1

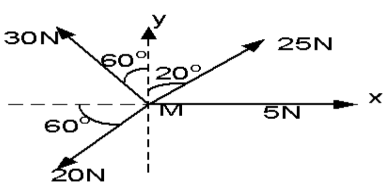
b



Three forces of 3.5N, 4.0N and 2.0N act at a point O as shown above. Find the resultant force. (4marks)
An[1.07N at 15.5° above the horizontal]

UNEB 2007 No 4

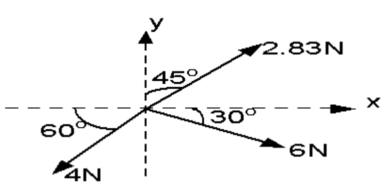
ii)



A body m of mass 6kg is acted on by forces of 5N, 20N, 25N and 30N as shown above. Find the acceleration of m [05 marks]
An[5.5ms^{-2}]

UNEB NOV/DEC 1998 No1

c)



Forces of 2.83N, 4.00N and 6.00N act on a particle O as shown above. Find the resultant force on the particle [06marks]

3.2.0: RELATIVE MOTION

It comprises of;
1-Relative velocity
2-Relative path

3.2.1: Relative velocity

This is the velocity a body would have as seen by an observer on another body. Suppose A and B are two moving bodies, the velocity of A relative to B is the velocity of A as it appears to an observer on B.

It's denoted by $_AV_B = V_A - V_B$

Note that $_AV_B \neq {_BV_A}$ since $[_BV_A = V_B - V_A]$

There are two methods used in calculations

- Geometric method
- Vectorial method

1. Vector method

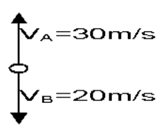
Find component of velocity for each object separately

Therefore $_AV_B = V_A - V_B$

Example

1. Particle A is moving due to north at $30ms^{-1}$ and particle B is moving due south at 20m/s. find the velocity of A relative to B.

Solution



$_AV_B = V_A - V_B$

$_AV_B = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ -20 \end{pmatrix} = \begin{pmatrix} 0 \\ 50 \end{pmatrix}$

$/_AV_B/ = \sqrt{0^2 + 50^2}$ = 50m/s due north

2. A cruiser is moving at 30km/hr due north and a battleship is moving at 20km/hr due north, find the velocity of the cruiser relative to the battleship.

Solution

$V_C = \begin{pmatrix} 0 \\ 30 \end{pmatrix}$ $V_B = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

$_CV_B = V_C - V_B$

$_CV_B = \begin{pmatrix} 0 \\ 30 \end{pmatrix} - \begin{pmatrix} 0 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ due north

$/_CV_B/ = \sqrt{0^2 + 10^2}$

$_CV_B$ =10km/h due north

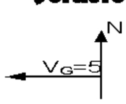
3. A particle A has a velocity of 4i+6j-5k (m/s) while particle B has a velocity of -10i-2j+6k (m/s). find the velocity of A relative to B

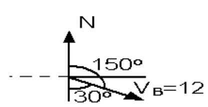
Solution

$$_AV_B = V_A - V_B = \begin{pmatrix} 4 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} -10 \\ -2 \\ 6 \end{pmatrix} = \begin{pmatrix} 14 \\ 8 \\ -11 \end{pmatrix} ms^{-1}$$

4. A boy runs at 5km/h due west and a girl runs 12km/hr at a bearing of 150°. Find the velocity of the girl relative to the boy.

Solution

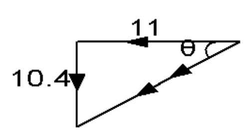




$_GV_B = V_G - V_B$

$_GV_B = \begin{pmatrix} -5 \\ 0 \end{pmatrix} - \begin{pmatrix} 12Sin30 \\ -12Cos30 \end{pmatrix} = \begin{pmatrix} -11 \\ -10.4 \end{pmatrix}$

$/_GV_B/ = \sqrt{(-11)^2 + (-10.4)^2}$ = 15.14km/hr

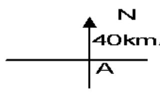


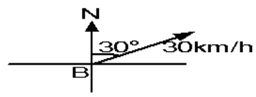
$\theta = tan^{-1}\left(\frac{10.4}{11}\right) = 43.4°$

Relative velocity is 15.14km/hr at 43.4° below the horizontal.

5. Plane A is flying due north at 40km/hr while plane B is flying in the direction N30°E at 30km/hr. Find the velocity of A relative to B.

Solution

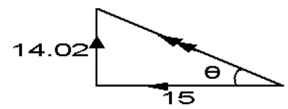




$_AV_B = V_A - V_B$

$_AV_B = \begin{pmatrix} 0 \\ 40 \end{pmatrix} - \begin{pmatrix} 30Sin30 \\ -30Cos30 \end{pmatrix} = \begin{pmatrix} -15 \\ 14.02 \end{pmatrix}$

$/_AV_B/ \; = \sqrt{(-15)^2 + (14.02)^2} \; = 20.53$km/hr

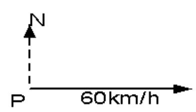


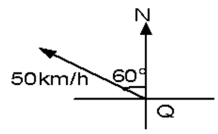
$\theta = tan^{-1}\left(\frac{14.02}{15}\right) = 43.07°$

The relative velocity is 20.53 at N46.93°W

6. Ship P is steaming at 60km/hr due east while ship Q is steaming in the direction N60°W at 50km/hr. Find the velocity of P relative to Q.

Solution



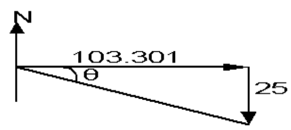


$V_p = \begin{pmatrix} 60 \\ 0 \end{pmatrix}$ $\quad V_q = \begin{pmatrix} -50Sin60 \\ 50Cos60 \end{pmatrix}$

$_pV_q = V_p - V_q$

$_pV_q = \begin{pmatrix} 60 \\ 0 \end{pmatrix} - \begin{pmatrix} -50Sin60 \\ 50Cos60 \end{pmatrix} = \begin{pmatrix} 103.301 \\ -25 \end{pmatrix}$

$/pVq/ \; = \sqrt{(103.301)^2 + (-25)^2} = 106.3$km/hr



$\theta = tan^{-1}\left(\frac{25}{103.301}\right) = 13.6°$

Direction $S(90 - 13.6)°E$

Relative velocity is 106.3km/hr at S76.4°E

7. To a cyclist riding due north at 40km/hr, a steady wind appears to blow from west at 30km/hr. find the true velocity of the wind.

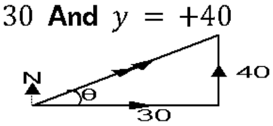
Solution

$V_c = \begin{pmatrix} 0 \\ 40 \end{pmatrix}$ $\quad _wV_c = \begin{pmatrix} 30 \\ 0 \end{pmatrix}$ $\quad V_w = \begin{pmatrix} x \\ y \end{pmatrix}$

$V_c = V_w - V_c$

$\begin{pmatrix} 30 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 40 \end{pmatrix}$

$x = 30$ And $y = +40$



$V\,w = \begin{pmatrix} 30 \\ 40 \end{pmatrix}$

$V_w = \sqrt{30^2 + 40^2} = 50$km/hr

$\theta = tan^{-1}\left(\frac{40}{30}\right) = 53.13°$

Direction $N(90 - 53.13)°E$

N36.87°E

Trial:3

1. Car A is moving East wards at 20m/s and car B is moving Northwards at 10m/s. find the
   i) Velocity of A relative to B **An [10$\sqrt{5}$ m/s]**
   ii) Velocity of B relative to A **An [10$\sqrt{5}$ m/s]**

2. In EPL football match, a ball is moving at 5m/s in the direction of N45°E and the player is running due north at 8m/s. Find the velocity of the ball relative to the player.
An[5.69m/s at S38.38°E].

3. A ship is sailing south East at 20km/hr and a second ship is sailing due west at 25km/hr. Find the magnitude and direction of the velocity of the first ship relative to the second. **An [41.62km/hr at S70.13°E]**

4. On a particular day wind is blowing N30°E at a velocity of 4m/s and a motorist is driving at 40m/s in the direction of S60°E
   a) Find the velocity of the wind relative to motorist **An [40.2m/s at N54.28°W]**
   b) If the motorist changes the direction maintaining his speed and the wind appears to blow due East. What is the new direction of the motorist? **An[N85.03°W]**

3.2.2: RELATIVE PATH

Consider two bodies A and B moving with V_A and V_B from points with position vectors R_A and R_B respectively.

Position of A after time t is

R_{At}=OA + t x V_A

Position of B after time t is

R_{Bt}= OB + t x V_B

Relative path

${}_AR_B = R_{At} - R_{Bt}$

${}_AR_B = (OA + tV_A) - (OB + tV_B)$

${}_AR_B = (OA - OB) + t(V_A - V_B)$

${}_AR_B = (OA - OB) + t({}_AV_B)$

EXAMPLE

1. A car A and B are moving with their respective velocities $2i - j$ and $i + 3j$, if their position vectors are $4i + j$ and $2i - 3j$ respectively. Find the path of A relative to B

i) At any time t

ii) At t=2s

Solution

i) $V_A = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $V_B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

OA = $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ OB= $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

${}_AR_B$= (OA-OB) + t(${}_AV_B$)

${}_AR_B = \left[\begin{pmatrix} 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \end{pmatrix}\right] + t\left[\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}\right]$

${}_AR_B = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

ii) When t=2 ${}_AR_B = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

${}_AR_B = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -8 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

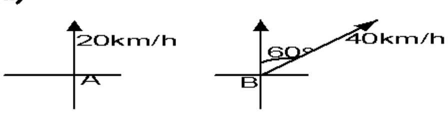
2. Two ships A and B move simultaneously with velocities 20km/hr and 40km/hr respectively. Ship A moves in the northern directions while ship B moves in N60°E. Initially ship B is 10km due west of A. determine

a) The relative velocity of A to B

b) The relative path of A to B

Solution

a)



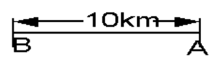
$V_A = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$ $V_B = \begin{pmatrix} 40Sin60 \\ 40Sin60 \end{pmatrix}$

${}_AV_B = V_A - V_B$

${}_AV_B = \begin{pmatrix} 0 \\ 20 \end{pmatrix} - \begin{pmatrix} 40Sin60 \\ 40Sin60 \end{pmatrix} = \begin{pmatrix} -34.64 \\ 0 \end{pmatrix}$

${}_AV_B$=34.64km/hr

b)



${}_AR_B = (OA - OB) + t({}_AV_B)$

OB= $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ OA= $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

${}_AR_B = \left[\begin{pmatrix} 10 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] + t\begin{pmatrix} -34.64 \\ 0 \end{pmatrix}$

${}_AR_B = \begin{pmatrix} 10 \\ 0 \end{pmatrix} + t\begin{pmatrix} -34.64 \\ 0 \end{pmatrix}$

3.2.3: SHORTEST DISTANCE AND TIME TO SHORTEST DISTANCE [DISTANCE AND TIME OF CLOSEST APPROACH]

When two particles are moving simultaneously with specific velocities, time will come when they are closest to each other **without** colliding

Numerical calculations

There three methods used

- Geometrical
- Vector
- Differential

1. Vector

Consider particles A and B moving with velocities V_A and V_B from point with positions vectors OA and OB respectively.

Then ***shortest distance*** **d** $= /{}_A R_B /$

For minimum distance to be attained then ${}_A V_B \cdot {}_A R_B = 0$ This gives the time

Or time $= \frac{/AB \ . \ AVB/}{/AVB/^2}$ Where **AB.** ${}_A V_B$ is a dot product

2. Differential

The minimum distance is reached when $\frac{d}{dt} /{}_A R_B /^2 = 0$ This gives the time

Minimum distance **d** $= /{}_A R_B /$

EXAMPLE

1. A particle P starts from rest from a point with position vector $2j + 2k$ with a velocity $(j + k)$m/s. A second particle Q starts at the same time from a point whose position vector is $-11i - 2j - 7k$ with a velocity of $(2i + j + 2k)$m/s . Find;
   - i) The shortest distance between the particles
   - ii) The time when the particles are closest together
   - iii) How far each has travelled by this time

Solution:

Method I vector

i) OP $= \begin{pmatrix}0\\2\\2\end{pmatrix}$ Vp $= \begin{pmatrix}0\\1\\1\end{pmatrix} m/s$

$OQ = \begin{pmatrix}-11\\-2\\-7\end{pmatrix}$ $V_Q = \begin{pmatrix}2\\1\\2\end{pmatrix}$m/s

${}_P V_Q = V_P - V_Q$

${}_P V_Q = \begin{pmatrix}0\\1\\1\end{pmatrix} - \begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}-2\\0\\-1\end{pmatrix}$

${}_P R_Q = (OP - OQ) + ({}_P V_Q)t$

$\text{PRQ} = \left[\begin{pmatrix}0\\2\\2\end{pmatrix} - \begin{pmatrix}-11\\-2\\-7\end{pmatrix}\right] + \begin{pmatrix}-2\\0\\-1\end{pmatrix} t$

${}_P R_Q = \begin{pmatrix}11\\4\\9\end{pmatrix} + \begin{pmatrix}-2\\0\\-1\end{pmatrix} t$

For minimum distance

${}_P V_Q . {}_P R_Q = 0$

$\begin{pmatrix}-2\\0\\-1\end{pmatrix} . \begin{pmatrix}11-2t\\4\\9-t\end{pmatrix} = 0$

$-22 + 4t + 0 - 9 + t = 0$

$t = \frac{31}{5}$ $\therefore$ $t = 6.2s$

ii) Shortest distance d=$/{}_P R_Q /$

${}_P R_Q = \begin{pmatrix}11\\4\\9\end{pmatrix} + \begin{pmatrix}-2\\0\\-1\end{pmatrix} t$

t=6.2

${}_P R_Q = \begin{pmatrix}11\\4\\9\end{pmatrix} + \begin{pmatrix}-2\\0\\-1\end{pmatrix} 6.2 = \begin{pmatrix}-1.4\\4\\2.8\end{pmatrix}$

$/{}_P R_Q / = \sqrt{(-1.4)^2 + 4^2 + 2.8^2}$

$/{}_P R_Q /$= **5.08m**

iii) How far each has travelled

R_P = OP +$V_P t$

$R_P = \begin{pmatrix}0\\2\\2\end{pmatrix} + \begin{pmatrix}0\\1\\1\end{pmatrix} 6.2 = \begin{pmatrix}0\\8.2\\8.2\end{pmatrix}$

$/R_P/ = \sqrt{0^2 + 8.2^2 + 8.2^2}$=**11.6m**

R_Q = OQ +$V_Q t$

$R_Q = \begin{pmatrix}-11\\-2\\-7\end{pmatrix} + \begin{pmatrix}2\\0\\1\end{pmatrix} 6.2 = \begin{pmatrix}1.4\\4.2\\5.4\end{pmatrix}$

$/R_Q/ = \sqrt{1.4^2 + 4.2^2 + 5.2^2}$= **6.8m**

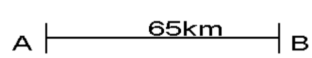
2. Initially two ships A and B are 65km apart with B due East of A. A is moving due East at 10km/hr and B due south at 24km/hr. the two ships continue moving with these velocities. Find the least distance between the ships in the subsequent motion and the time taken to the nearest minute for such a situation to occur.

Solution

least distance d = $/{}_A R_B/$

For least distance $({}_A V_B . {}_A R_B)$ = 0

But ${}_A V_B = \begin{pmatrix}10\\24\end{pmatrix}$



A $= \begin{pmatrix}0\\0\end{pmatrix}$ B= $\begin{pmatrix}65\\0\end{pmatrix}$

${}_{A}R_{B} = (OA - OB) + {}_{A}V_{B}t$

$= \left[\begin{pmatrix}0\\0\end{pmatrix} - \begin{pmatrix}65\\0\end{pmatrix}\right] + t\begin{pmatrix}10\\24\end{pmatrix}$

${}_{A}R_{B} = \begin{pmatrix}-65+10t\\24t\end{pmatrix}$

${}_{A}V_{B} \cdot {}_{A}R_{B} = 0$

$\begin{pmatrix}10\\24\end{pmatrix} \cdot \begin{pmatrix}-65+10t\\24t\end{pmatrix} = 0$

$-650 + 100t + 576t = 0$

$t = \frac{650}{676} \quad \therefore \; t = 0.96hrs$

Trial:4

1. A ship A is 8km due North of Ship B, ship A is moving at $150kmh^{-1}$ due west while B is moving at 200km/hr due N30°W. After what time will they be nearest together and how far apart will they be. **An(2.22km, 0.043hrs)**
2. The point p is 50km west of q. Two air crafts A and B fly simultaneously from p and q velocities are 400km/hr N50°E and 500km/hr N20°W respectively. Find;
   (i) The closest distance between the air crafts
   (ii) The time of flight up to this point **An(20.35km, 5.24 minutes)**
3. Ship A steams North-west at 60km/hr whereas B steams southwards at 50km/hr, initially ship B was 80km due north of A. find;
   (i) The velocity of A relative to B
   (ii) The time taken for the shortest distance to be reached
   (iii) The shortest distance between A and B.
   An(101.675km/hr at N24.7°W, 42.9minutes, 33.382km)

3.3.0: Motion of bodies with different frames of reference

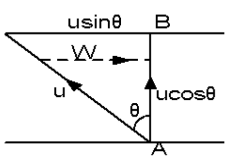
It involves crossing the river and flying space

3.3.1: Crossing the river

There are three cases to consider when crossing a river

a. Case I (shortest route)

If the water is not still and the boat man wishes to cross **directly opposite** to the starting point.
In order to cross point A to another point B directly opposite A (perpendicularly), then the course set by the boat must be upstream of the river.



u is the speed of the boat in still water,
w is the speed of the running water

At point B: $u\sin\theta = w$

$\sin\theta = \frac{w}{u}$

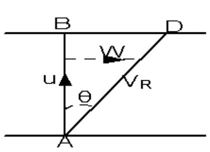
$\theta = \sin^{-1}\frac{w}{u}$

θ is the direction to the vertical but the direction to the bank is $(90-\theta)°$

Time taken $= \frac{AB}{u\cos\theta}$

b. Case II. The shortest time/as quickly as possible

If the boat man wishes to cross the river as quickly as possible, then he should steer his boat directly from A to B as shown. The river pushes him down stream.



Time to cross the river $t = \frac{AB}{u}$

Distance covered downstream is $= wxt$

Or distance downstream $= w\frac{AB}{u}$

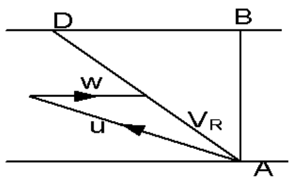
$\tan\theta = \frac{w}{u} \qquad \theta = \tan^{-1}\frac{w}{u}$

The resultant velocity downstream V_R

$V_R^2 = W^2 + u^2$

$V_R = \sqrt{w^2 + u^2}$

C. Case III



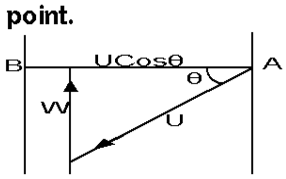
Resultant velocity $\overrightarrow{V_R} = \overrightarrow{W} + \overrightarrow{U}$

EXAMPLES

1. A river with straight parallel bank 400m apart flows due north at 4km/hr. Find the direction in which a boat travelling at 12km/hr must be steered in order to cross the river from East to West along the course perpendicular to the banks. Find also the time taken to cross the river.

Solution

Hint. Since the course is perpendicular to the bank, then it requires crossing directly to the opposite point.



W=4km/hr U=12km/hr

AB=400m=0.4km

Sinθ=$\frac{w}{u}$ θ=$Sin^{-1}\frac{4}{12}$ θ=19.47°

The direction is (90-19.47) to the bank.

Direction is 70.53° to the bank

Time taken =$\frac{AB}{uCos\theta}$

Time taken =$\frac{0.4}{12Cos19.47}$= 0.035hrs

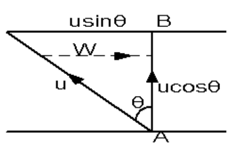
Time =2.1 minutes

2. A man who can swim at 6km/hr in still water would like to swim between two directly opposite points on the banks of the river 300m wide flowing at 3km/hr. Find the time he would take to do this.

Solution

U=6km/hr W= 3km/hr

AB=300m AB = 0.3km



$Sin\theta = \frac{w}{u}$

$\theta = Sin^{-1}\left(\frac{3}{6}\right) = 30°$

Time taken $= \frac{AB}{uCos\theta}$

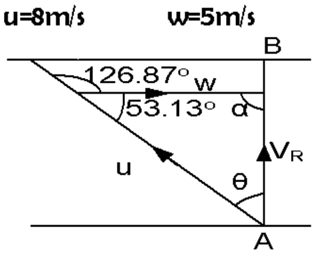
Time taken $= \frac{0.3}{6Cos30}$ = 0.058hrs=3.46minute

He must swim at 30° to AB in order to cross directly and it will take 3.46minutes

3. A man who can swim at 8m/s in still water crosses a river by steering at an angle of 126.87° to the water current. If the river is 75m wide and flows at 5m/s, find;
   (i) The velocity with which the person crosses the river
   (ii) The time he takes to do this

Solution

u=8m/s w=5m/s AB=75m



α is not 90°

Using cosine rule

$V_R^2 = 8^2 + 5^2 - 2\,u.wcos53.13$

$V_R = \sqrt{8^2 + 5^2 - 2\,x8x5cos53.13}$

$V_R = 6.4m/s$

The person crosses with 6.4m/s.

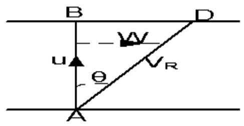
ii) Time taken =$\frac{AB}{uCos\theta}$

But $V_R = Ucos\theta$

Time $= \frac{75}{6.4}$ = 11.72 seconds

4. A man who can swim at 2m/s at in still water wishes to swim across a river 120m wide as quickly as possible. If the river flows at 0.5m/s, find the time the man takes to cross and how far down streams he travels.

Solution



$U = 2m/s \quad w = 0.5m/s \quad AB = 120m$

$t = \frac{AB}{u} = \frac{120}{2} = 60s$

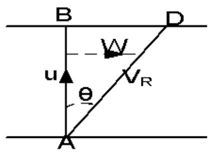
Distance downstream $= wt = 0.5x60$

Distance downstream $= 30m$

5. A boat can travel at 3.5m/s in still water. A river is 80m wide and the current flows at 2m/s, calculate
   a) The shortest time to cross the river and the distance downstream that the boat is carried.
   b) The course that must be set to apoint exactly opposite the starting point and the time taken for crossing

Solution

a) U=3.5m/s, w=2m/s AB=80m

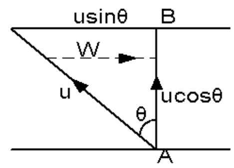


Shortest time $t = \frac{AB}{u} = \frac{80}{3.5}$ = 22.95

Distance downstream BD=wt$= 2x22.9$

Distance downstream BD=45.8m

b. U=3.5m/s, w=2m/s, AB=80



$Sin\theta = \frac{w}{u}$

$\theta = Sin^{-1}\left(\frac{2}{3.5}\right)$ =34.8°

The course must be 34.8° to AB.

Time taken $= \frac{AB}{uCos\theta} = \frac{80}{3.5Cos34.8} = 27.8s$

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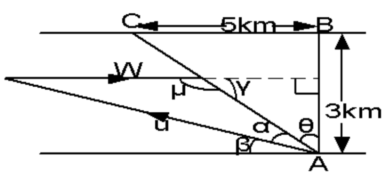
6. A boat crosses a river 3km wide flowing at 4m/s to reach a point on the opposite bank 5km upstream. The boat's speed in still water is 12m/s. Find the direction in which the boat must be headed. (04marks)

Solution

In order for a boat to cross to a point C upstream on the opposite bank then the course set must be such that the resultant velocity of the boat is along AC upstream.

U=12m/s, w=4m/s, AB=3km, AC=5km



$\tan\theta = \frac{5}{3}$ $\quad \theta$=59.04°

But $\gamma + \theta$=90°

$\gamma = 90 - 59.04$

γ =30.96°

But $\mu + \gamma$ =180°

μ +30.96°=180°

μ =180°-30.96°

μ=149.04°

Also using sin rule $\frac{w}{sin\alpha} = \frac{u}{sin\mu}$

$$\frac{4}{sin\alpha} = \frac{12}{sin149.04}$$

$\alpha = sin^{-1}\left(\frac{4sin149.04}{12}\right)$

α=9.87°

But $\beta + \alpha + \theta$=90°

β +9.87+59.04=90°

β =21.09°

The boat must be headed at 21.09° to the river bank upstream

Trial:4

1. A man who an row at 0.9m/s in still water wishes to cross the river of width 1000m as quickly as possible. If the current flows at a rate of 0.3m/s. Find the time taken for the journey. Determine the direction in which he should point the boat and position of the boat where he lands **An [1111.11s, 71.57° to the bank, 333.33 downstream]**

2. A man swims at $5kmh^{-1}$ in still water. Find the time it takes the man to swim across the river 250m wide, flowing at $3kmh^{-1}$, if he swims so as to cross the river;
   (i) By the shortest route **An [178.6s]**
   (ii) In the quickest time **An[217.4s]**
3. A boy can swim in still water at 1m/s, he swims across the river flowing at 0.6m/s which is 300m wide, find the time he takes;
   (i) If he travels the shortest possible distance
   (ii) If he travels as quickly as possible and the distance travelled downstream. **[375s,180m]**
4. Rain drops of mass $5x10^{-7}kg$ fall vertically in still air with a uniform speed of 3m/s. if such drops are falling when a wind is blowing with a speed 2m/s,
   (i) what is the angle which the paths of the drops make with the vertical
   (ii) what is the kinetic energy of a drop **[33.7^{0},$3.25x10^{-6}$J]**
5. A boy wishes to swim across a river 100m wide as quickly as possible. The river flows at 3km/hr and the boy can swim at 4km/h in still water. Find the time that the boy takes to cross the river and how far downstream he travels. **An [90s,75m].**

CHAPTER 4: NEWTON'S LAWS OF MOTION

LAW I : Everybody continues in its state of rest or uniform motion in **a straight line** unless acted upon by an external force.

This is sometimes called the law of **inertia**

Definition

Inertia is the reluctance of a body to start moving once its at rest or to stop moving if its already in motion.

Explain why a passenger jerks forward when a fast moving car is suddenly stopped.

Passengers jerk forward because of inertia. When the car is suddenly stopped, the passenger tends to continue in uniform motion in a straight line because the force that acts on the car does not act on the passenger

LAW II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

Consider a mass m moving with velocity u. If the mass is acted on by a force F and its velocity changes to v;

By Newton's law of motion

F $\alpha \frac{mv-mu}{t} = \frac{k(mv-mu)}{t} = km\frac{(v-u)}{t} = kma$

Since $a = \frac{v-u}{t}$

When $F = 1N, m = 1kg$ and $a = 1ms^{-2}$

$1 = kx1x1$

$k = 1$

$\boxed{F = ma}$

Note: F must be the resultant force

LAW III: To every action there is an equal but opposite reactions.

$$F_1 = -F_2$$

Example of 3rd law of motion

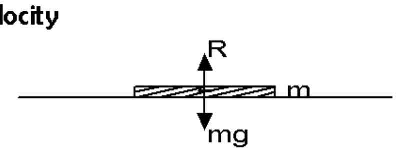
- A gun moves backwards on firing it.
- A ball bounces on hitting the ground.

Rocket engine propulsion

Fuel is burnt in the combustion chamber and exhaust gases are expelled at a high velocity. This leads to a large backward momentum. From conservation of momentum an equal forward momentum is gained by the rocket, due to continuous combustion of fuel there is a change in the forward momentum which leads to the thrust hence maintaining the motion of the rocket

4.1.0: IDENTIFICATION OF FORCES AND THE APPLICATION OF NEWTON'S LAWS

1. Consider a body of mass **m** placed on either a stationary platform or a platform moving at a constant velocity

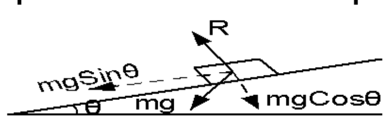


R is normal reaction

Mg is gravitational pull [weight]

$R = mg$ since (a=0) constant velocity

2. Mass m placed on a smooth inclined plane of angle of inclination θ



R = mgCosθ

NB:

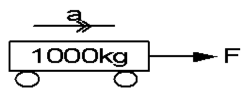
- All objects placed on, or moving on an inclined plane experience a force $mgsin\theta$ **down** the plane. [It doesn't matter what direction the body is moving]
- If the plane is **rough** the body experiences a frictional force whose direction is opposite to the direction of motion.

Examples:

1. A car of mass 1000kg is accelerating at $2ms^{-2}$.
   (i) What resultant force acts on the car?.
   (ii) If the resistance to the motion is 1000N, what force is due to the engine?

Solution

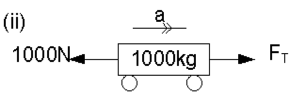
(i)



$F = ma = 1000x2 = 2000N$

Resultant force is 2000N

(ii)



The resistance force should act in opposite direction to the force due to the engine

$F_T - 1000 = ma$

$F_T - 1000 = 1000x2$

$F_T = 3000N$

Force due to the engine is 3000N

2. A car moves along a level road at a constant velocity of 22m/s. If its engine is exerting a forward force of 2000N, what resistance is the car experiencing

Solution

Using $F = ma$

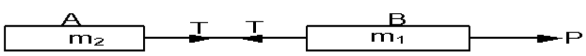
$2000 - R_1 = ma$

But a = 0 since it moves with constant velocity

$2000 - R_1 = 0$

$R_1 = 2000N$

3. Two blocks A and B connected as shown below on a horizontal friction less floor and pulled to the right with an acceleration of $2ms^{-2}$ by a force P, if $m_1 = 50kg$ and $m_2 = 10kg$. what are the values of T and P



Solution

Using $F = ma$

For m_1: $P - T = 50x2 = 100$…….[1]

For m_2 $T = 10x2 = 20N$

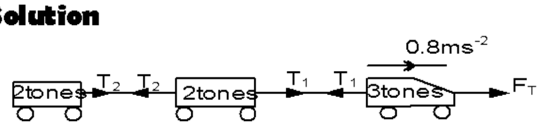
Put into equation (1) $P - T = 100$

$P - 20 = 100$

$P = 120N$

4. A Lorry of 3 tones pulls 2 trailers each of mass 2 tones along a horizontal road, if the lorry is accelerating at $0.8ms^{-2}$, calculate
   a) Net force acting on the whole combination
   b) The tension in the coupling between the lorry and 1st trailer.
   c) The tension in the coupling between the 1st and 2nd trailer.

Solution



For the lorry: $F_T - T_1 = 3000x0.8 = 2400 \ldots..(1)$

For 1st trailer: $T_1 - T_2 = 2000x0.8 = 1600 \ldots..(2)$

For 2ND trailer: $TT_2 = 2000x0.8 = 1600N$

Put into [2]: $T_1 - T_2 = 1600$

$T_1 - 1600 = 1600$

$T_1 = 3200N$

Put into [1] $F - T_1 = 2400$

$F - 3200 = 2400$

$F = 5600N$

Exercise: 5

1. A large card board box of mass 0.75kg is pushed across a horizontal floor by a force of 4.5N. the motion of the box is opposed by a frictional force of 1.5N between the box and the floor , and an air resistance force given by kv^2 where $k = 6.0x10^{-2}kgm^{-1}$and v is the speed of the box in m/s. calculate;
   (a) The acceleration of the box
   (b) Its speed **An(4.0m/s², 7.1m/s)**
2. A stone of mass 500g is thrown with a velocity of $15ms^{-1}$ across the frozen surface of a lake and comes to rest in 40m. what is the average force of the friction between the stone and the ice
3. A 5000kg engine pulls a train of 5 trucks, each of 2000kg along a horizontal track. If the egnie exerts a force of 50000N and frictional resistance is 5000N calculate;
   (a) The net accelerating force
   (b) The acceleration of the train

(c) The force of truck 1 on truck 2

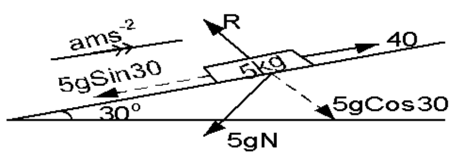
4. A dummy is used in a test crash to test the suitability of the seat belt. If the dummy had a mass of 65kg and it was brought to rest in a distance of 65cm form a velocity of 12m/s, calculate
   (a) the mean deceleration during the crash
   (b) The average force exerted on the dummy during the crash
5. A box of 50kg is pulled up from a ship with an acceleration of $1ms^{-2}$ by a vertical rope attached to it.
   i. Find the tension on the rope.
   ii. What is the tension in the rope when the box moves up with a uniform velocity of $1ms^{-1}$ ($g=9.8ms^{-2}$) **An [540N, 490N]**
6. A lift moves up and down with an acceleration of $2ms^{-2}$. In each case, calculate the reaction of the floor on a man of mass 50kg standing in the lift. (take g = $9.8ms^{-1}$) **An[590N, 390N]**

Motion on inclined planes

Example

1. A body of mass 5kg is pulled up a smooth plane inclined at 30° to the horizontal by a force of 40N acting parallel to the plane. Find
   a) Acceleration of the body
   b) Force exerted on the body by the plane

Solution



a) Resolving parallel to the plane: $F = ma$

$$40 - 5gsin30 = ma$$

$$40 - 5x9.81sin30 = 5xa$$

$$a = 3.095ms^{-2}$$

b) Force exerted on the body by the plane is the normal reaction

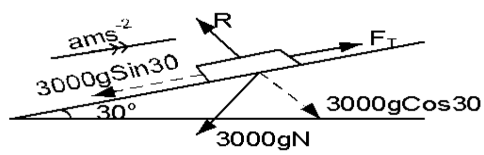
$$R = 5gcos30 = 5x9.81cos30 = 42.4N$$

2. A lorry of mass 3 tones travelling at 90km/hr starts to climb an incline of 1 in 5. Assuming the tractive pull between its tyres and the road remains constant and that its velocity reduces to 54km/h in a distance of 500m. Find the tractive pull

Solution

$$u = 90km/h = \frac{90x1000}{3600} = 25ms^{-1}$$

$$v = 54km/h = \frac{54x1000}{3600} = 15ms^{-1}$$



Resolving along the plane

$$F_T - 3000gsin\theta = 3000a$$

$$F_T - 3000x9.81x\frac{1}{5} = 3000a$$

$F - 5886 = 3000a$……………….(i)

But $v^2 = u^2 + 2as$

$$15^2 = 25^2 + 2ax500$$

$$a = -0.4ms^{-2}$$

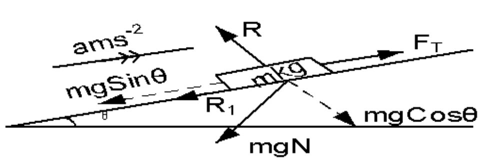
put into (i) $F - 5886 = 3000a$

$$F = -3000x0.4 + 5886 = 4686N$$

The tractive force is 4686N

3. A train travelling uniformly at 72km/h begins an ascent on 1 in 75. The tractive force which the engine exerts during the ascent is constant at 24.5kN, the resistance due to friction and air is also constant at 14.7kN, given the mass of the whole train is 225 tones. Find the distance a train moves up the plane before coming to rest.

Solution



1 in 75 means $sin\theta = \frac{1}{75}$ ∴ $\theta = 0.76°$

resistance force: $R_1 = 14.7kN$

tractive force : $F_T = 24.5kN$

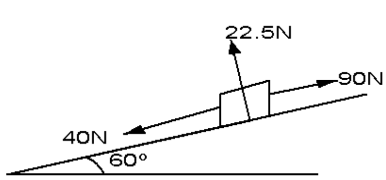
$$F_T - (mgsin\theta + R_1) = ma$$

$$24500-(225000x9.81x\frac{1}{75}+14700) = 22500a$$
$$a = -0.087ms^{-2}$$
its deceleration $= 0.087ms^{-2}$
$v^2 = u^2 + 2as$ [$v = om/s$ comes to rest]

$$u = 72km/h = \frac{72x1000}{3600} = 20ms^{-1}$$
$$0^2 = 20^2 + 2(-0.087)s$$
$$-400 = -0.174s$$
$$S = 2298.85m$$

Exercise:6

1. The resistance to the motion of the train due to friction is equal to 1/160 of the weight of the train, if the train is travelling on a level road at $72kmh^{-1}$ and comes to the foot of an incline of 1 in 150 and steam is then turned off, how far will the train go up the incline before it comes to rest. **An(1579.99m)**
2. 12m length of the slope. If the truck starts from the bottom of the slope with a speed of 18km/h, how far up will it travel before coming to rest **An(71.43m).**
3. A car of 1 tonne accelerates from 36kmh to $72\ kmh^{-1}$ while moving $0.5kmh^{-1}$ up a road inclined at an angle of α to the horizontal, where $sin\alpha = \frac{1}{20}$. If the total resistive force to its motion is 0.3kN, find the driving force of the car engine **An(1009N).**
4. A railway truck of mass 6.0 tonnes moves with an acceleration of $0.050ms^{-2}$ down a track which is inclined to the horizontal at an angle α where $sin\alpha = \frac{1}{120}$. Find the resistance to motion **An(2.0x10^{2}N).**
5. A body of mass 5.0kg is pulled along a smooth horizontal ground by means of force of 40N acting at 60° above the horizontal. Find
   (a) Acceleration of the body
   (b) Force the body exerts on the ground **An(4.0ms^{-2}, 15.4N).**
6. A railway engine of mass 100 tones is attached to a line of truck of total mass 80 tones. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train
   (a) has an acceleration of $0.020ms^{-2}$
   (b) Is moving at constant velocity **An(25.6kN).**
7. A bullet of mass $8.00x10^{-3}$kg moving at $320ms^{-1}$ penetrates a target to a depth of 16.0mm before coming to rest. Find the resistance offered by the target, assuming it to be uniform. **An(1.6kN, 0N).**
8. A body of mass 3.0kg slides down a plane which is inclined at 30° to the horizontal. Find the acceleration no of the body , if:
   (a) The plane is smooth
   (b) There is a frictional resistance of 9.0N **An(5.0ms^{-2}, 2.0ms-^{2}).**
9. A car of mass 1000kg tows a caravan of mass 600kg up a road which rises 1m vertically for every 20m of its length. There are constant frictional resistance of 200N and 100N to the motion of the car and to the motion of the caravan respectively. The combination has an acceleration of $1.2ms^{-2}$ with the engine exerting a constant driving force. Find
   (a) Driving force
   (b) Tension in the tow- bar **An(3.02kN, 1.12kN).**
10. A 25kg block rests at the top of a smooth plane whose length is 2.0m and whose height at elevated end is 0.5m. how long will it take for the block to slide to the bottom of plane when released **An(1.258s).**
11.



Three forces act on a block as shown, the block is placed on a smooth plane inclined at 60° calculate;
   a) Acceleration of the block up the plane
   b) Gain in kinetic energy in 5s after moving from rest **An(1.5ms^{-2}, 140.625J)**

4.1.1: MOTION OF CONNECTED PARTICLES

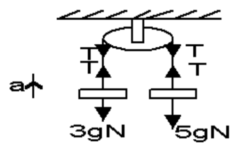
When two particles are connected by a light inextensible string passing over a smooth pully and allowed to move freely, then as long as the string is tight, the following must be observed.

- Acceleration of one body in general direction of motion is equal to the acceleration of the other
- The tension T in the string is constant.

Examples

1. Two particles of masses 5kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
   (i) Acceleration of the particles
   (ii) The tension in the string
   (iii) The force on the pulley

Solution



Using $F = ma$

For 5kg mass: $5g - T = 5a$...............(i)

For 3kg mass: $T - 3g = 3a$ ....(ii)

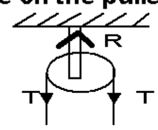
Adding (i) and (ii): $2g = 8a$

$a = \frac{2x9.81}{8} = 2.45ms^{-2}$

ii) $T - 3g = 3a$

$T = 3x2.45 + 3x9.81 = 36.78N$

iii) Force on the pulley

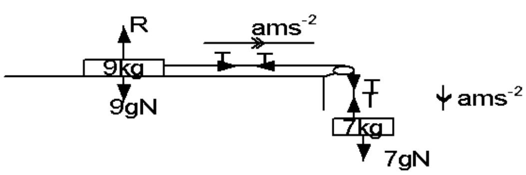


$R = 2T = 2x36.78 = 73.56N$

Force on the pulley is 73.56N

2. A mass of 9kg resting on a smooth horizontal table is connected by a light string passing over a smooth pulley at the edge of the table, to the pulley is a 7kg mass hanging freely; find
   (i) Common acceleration
   (ii) The tension in the string
   (iii) The force on the pulley in the system if its allowed to move freely.

Solution



Using $F = ma$

For 7kg mass: $7g - T = 7a$..............(i)

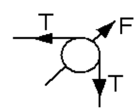
For 9kg mass: $T = 9a$.............................(ii)

Put (ii) into (i): $7g - 9a = 7a$

$a = \frac{7g}{16} = \frac{7x9.81}{16} = 4.292ms^{-2}$

(ii) Tension : $T = 9a = 9 \, x \, 4.292 = 38.63N$

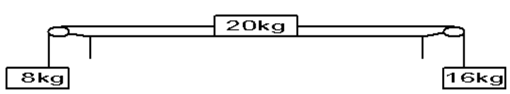
(iii) The force on the pulley



$F = \sqrt{T^2 + T^2} = T\sqrt{2} = 38.63\sqrt{2}$

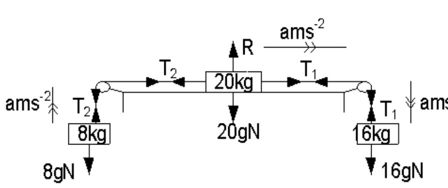
Force on the pulley $= 54.63N$

3.



The figure shows a block of mass 20 kg resting on a smooth horizontal table. Its connected by strings which pass over pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang vertically. Calculate;
   (i) Acceleration of 16kg mass
   (ii) Tension in each string
   (iii) Reaction on each pulley

Solution



Using $F = ma$

For 16kg mas: $16g - T_1 = 16a$..........[1]

For 20kg mas: $T_1 - T_2 = 20a$............[2]

For 8kg mass $T_2 - 8g = 8a$ ....[3]

Adding 1 and 2: $16g - T_2 = 36a$.............[x]

And (3) and (x): $8g = 44a$

$a = \frac{8x\,9.81}{44} = 1.784ms^{-2}$

ii) Tension in each string

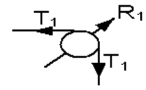
$16g - T_1 = 16a$

$T_1 = 16x9.81 - 16x1.784 = 128.416N$

$T_2 - 8g = 8a$

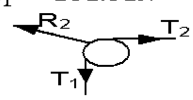
$T_2 = 8x1.784 + 8x9.81 = 92.752N$

iii) Reaction on each pulley



$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 128.416x\sqrt{2}$

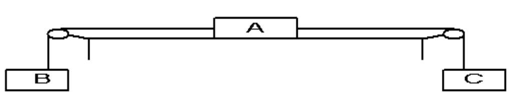
$R_1 = 181.61N$



$R_2 = T_2\sqrt{2} = 92.752\sqrt{2} = 131.171N$

Exercises:7

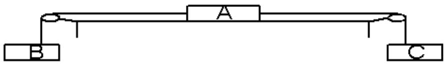
1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find;
   (i) Acceleration of the particles (ii) The tension in the string
   (iii) The force on the pulley **An(3.92ms^{-2} , 41.16N, 82.32N)**
2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find;
   (i) Acceleration of the particles (ii) The tension in the string
   (iii) Distance moved by the 6kg mass in the first 2 seconds of motion
   An(4.9ms^{-2} , 3N, 9.8m)
3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards
   (a) what is the tension in the section of the section of rope supporting the man
   (b) What is the acceleration of the bucket **An(807.06N, 1.73ms^{-2})**
4. Two particles of masses 20g and 30g are connected to a fine string passing over a smooth pulley, when released freely find;
   (i) Common acceleration (ii) The tension in the string
   (iii) The force on the pulley **An [1.962ms^{-2} , 0.235N ,0.471N]**
5. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate;
   a) The common acceleration of the masses **An[3.68m/s^2, 18.4N, 26N]**
   b) The tension in the string **c)** The force acting on the pulley
6. Two objects of mass 3kg and 5kg are attached to the ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3 kg mass touching the floor and the 5kg mass at 4m above the ground and then released, what is
   (i) The acceleration of the system **An(2.45ms^{-2}).**
   (ii) The tension of the cord **An(36.75N).**
   (iii) Time will elapse before the 5kg object hits the floor **An(1.81s).**
7.



The diagram shows a particle A of mass M = 2kg resting on a horizontal table. It is attached to particles B of m =5kg and C of m= 3kg by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string given that the surface of the table is rough and the coefficient of friction between the particle and the surface of the table is ½ **An[0.98ms^{-2}, 32.37N, 44.15N]**

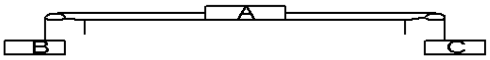
[Hint: friction force = coefficient of friction x normal reaction]

8.



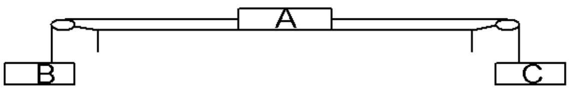
The diagram shows a particle A of mass $2kg$ resting on a rough horizontal table of coefficient of friction 0.5. It is attached to particles B of mass $5kg$ and C of mass $3kg$ by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string. **An[0.98ms^{-2}, 32.37N, 44.15N]**

9.



The diagram shows a particle A of mass $5kg$ resting on a rough horizontal table. It is attached to particles B of mass $3kg$ and C of mass $2kg$ by light inextensible strings hanging over light smooth pulleys. If the system is released from rest, body B descends with an acceleration of 0.28ms^{-2}, find the c coefficient of friction between the body A and the surface of the table **An[$\frac{1}{7}$]**

10.



The diagram shows a particle A of mass $10kg$ resting on a smooth horizontal table. It is attached to particles B of mass $4kg$ and C of mass $7kg$ by light inextensible strings hanging over light smooth pulleys. If the system is allowed to move from rest, find the common acceleration of the particle and the tension in each string. **An [1.4ms^{-2}, 44.8N, 58.8N]**

4.1.2: LINEAR MOMENTUM AND IMPULSE

Momentum is the product of mass and velocity of the body moving in a straight line

Momentum (p) = mass x velocity

$$\boxed{\vec{p} = m\vec{v}}$$

Momentum is a vector quantity

Definition

Linear momentum (p) is the product of the mass and the velocity of the body moving in a straight line.

IMPULSE

This is the product of the force and time for which the force acts on a body

i.e. Impulse (I) = Force(F) x time (t)

$$\vec{I} = \vec{F}t$$

The unit of impulse is Ns.

An impulse produces a change in momentum of a body. If a body of mass(m) has it velocity changed from u to v by a force F acting on it in time t, then from Newton's 2nd law.

$$F = \frac{mv - mu}{t}$$

$$Ft = mv - mu$$

$$I = Ft$$

$$I = mv - mu$$

Impulse = change in momentum

Example

1. A body of mass 5kg is initially moving with a constant velocity of 2ms^{-1}, when it experiences a force of 10N is 2s, find
   (i) The impulse given to the body by the force
   (ii) The velocity of the body when the force stops acting

Solution

$$I = ft = 10x2 = 20Ns$$

$$I = mv - mu$$

$$20 = 5v - 5x2$$

$$v = 6m/s$$

2. A girl of mass 50kg jumps onto the ground from a height of 2m. Calculate the force which acts on her when she lands

(i) As she bends her knees and stops within 0.2 s
(ii) As she keeps her legs straight and stops in 0.05s

Solution

i) $v^2 = u^2 + 2gs$
$v^2 = 0^2 + 2x9.81x2$
$v = \sqrt{39.24} = 6.3ms^{-1}$

Using $F = \frac{mv\text{-}mu}{t}$
$F = \frac{50(6.3\text{-}0)}{0.2} = 1575N$

ii) ii) $F = \frac{mv-mu}{t}$
$F = \frac{50(6.3\text{-}0)}{0.05} = 6300N$

3. Water leaves horse pipe at a rate of 5.0kgs^{-1} with a speed of 20ms^{-1} and is directed horizontally on a wall which stops it. Calculate the force exerted by the water on the wall.

Solution

Force due to water= $mass\ per\ second\ xvelocity\ change$
Force due to water= $5\ x(20-0) = 100N$

4. A horse pipe has a hole of cross-sectional area 50cm^2 and ejects water horizontally at a speed of 0.$3ms^{-1}$. If the water is incident on a vertical wall and its horizontal velocity becomes zero. Find the force the water exerts on the wall.

Solution

Force due to water= $mass\ per\ second\ xvelocity\ change$
Force due to water= $(areaxvelocityx\ density)xvelocity\ change = \rho Av^2$
Force due to water= $0.3x50x10^{-4}x1000\ x(0.3-0)$ =**0.45N**

5. A helicopter of mass 1.0x10^3kg hovers by imparting a downward velocity v to the air displaced by its rotating blades. The area swept pout by the blades is 80m^2. Calculate the value of v. (density of air=1.3kgm^{-3})

Solution

$$F = \rho Av^2$$
$$mg = \rho Av^2$$
$$1.0x10^3x9.81 = 80xvx1.3x(v-0)$$
$$1.0x10^3x9.81 = 104v^2$$
$$v = 9.8m/s$$

6. Sand falls onto a conveyor belt at a constant rate of 2kgs^{-1}. The belt is moving horizontally at 3ms^{-1}. Calculate
   (a) The extra force required to maintain the speed of the belt
   (b) Rate at which this force is dong work
   (c) The rate at which the kinetic energy of the sand increases

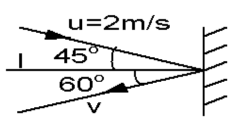
Solution

Force =mass per second $x$velocity change
$= 2x3 = 6N$
Rate of doing work =force x velocity change
$= 6x3 = 18Js^{-1}$

Rate of k.e $= \frac{1}{2}m\ x\ (velocity\ change)^2$
$= \frac{1}{2}x2x3^2 = 9Js^{-1}$

7. A ball of mass 0.25kg moving in a straight line with a speed of 2ms^{-1} strikes a vertical wall at an angle of 45° to the normal. The wall gives it an impulse in the direction of the normal and the ball rebounds at an angle of 60° to the normal. Calculate the magnitude of the impulse and the speed with which the ball rebounds.

Solution



Impulse $I = mv - mu$
$$I = m\left[\begin{pmatrix}-vCos60\\-vSin60\end{pmatrix} - \begin{pmatrix}2cos45\\-2sin45\end{pmatrix}\right]$$
$$I = \frac{1}{4}\left[\begin{pmatrix}\frac{-1}{2}v\\\frac{-\sqrt{3}}{2}v\end{pmatrix} - \begin{pmatrix}\sqrt{2}\\-\sqrt{2}\end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix}-\frac{v}{2}-\sqrt{2}\\-\frac{\sqrt{3}}{2}v+\sqrt{2}\end{pmatrix}$$

Since I is perpendicular to the wall then the vertical component is zero
$$-\frac{v}{2} - \sqrt{2} = 0$$
$$V = -2\sqrt{2}m/s$$
$$I = \frac{1}{4}\begin{pmatrix}-\frac{-2\sqrt{2}}{2}-\sqrt{2}\\-\frac{\sqrt{3}}{2}x-2\sqrt{2}+\sqrt{2}\end{pmatrix} = \frac{1}{4}\begin{pmatrix}0\\\sqrt{6}+\sqrt{2}\end{pmatrix} = \begin{pmatrix}0\\0.966\end{pmatrix}$$
$$I = 0.966Ns$$

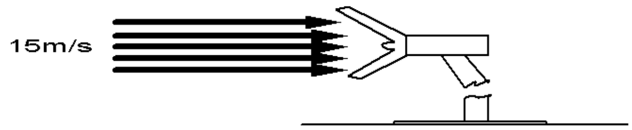
Exercises8

1. A horizontal jet of water leaves the end of a hose pipe and strikes a wall horizontally with a velocity of 20m/s. if the end of the pipe has a diameter of 2cm, calculate the force that will be exerted on the wall. **An(125.7N)**
2. Water emerges at $2ms^{-1}$ from a hose pipe and hits a wall at right angles. The pipe has across-sectional area of $0.03m^2$. calculate the force on the wall assuming that the water does not rebound.(density of water $1000kgm^{-3}$) **An(120N)**
3. Water is squirting horizontally at $4.0ms^{-1}$ from a burst pipe at a rate of $3.0kgs^{-1}$. The water strikes a vertical wall at right angles and runs down it without rebounding. Calculate the force the water exerts on the wall **An(12N)**
4. A machine gun fires 300 bullets per minute horizontally with a velocity of $500ms^{-1}$. Find the force needed to prevent the gun moving back-ward if the mass of each bullet $8.0x10^{-3}kg$ **An(20N)**
5. Coal is falling onto a conveyor belt at a rate of 540 tones every hour. The belt is moving horizontally at $2.0ms^{-1}$. Find the extra force required to maintain the speed of the belt **An($3.0x10^2$N)**
6. A helicopter of total mass 1000kg is able to remain in a stationary position by imparting a uniform downward velocity to a cylinder of air below it of effective diameter 6m. assuming the density of air to be 1.2kgm, calculate the downward velocity given to air **An($17.2ms^{-1}$)**
7. (a) The rotating blades of a hovering helicopter seeps out an area of radius 4.0m imparting a down ward velocity of $12ms^{-1}$ to the air displaced. Find the mass of the helicopter.(density of air $1.3kgm^{-3}$) **An(940kg)**

   (b) the sped of rotation of the blades of the helicopter is now increased so that the air has a down ward velocity of $13ms^{-1}$. Find the upward acceleration of the helicopter **An($1.7ms^{-2}$)**
8. Find the force exerted on each square meter of a wall which is at right angles to a wind blowing at $20ms^{-1}$. Assume that the air does not rebound.(density of air $1.3kgm^{-3}$) **An(520N)**
9. Hail stones with an average mass of 4.0g fall vertically and strike a flat roof at $12ms^{-1}$. In a period of 5.0minutes, 6000 hailstones fall on each square meter of roof and rebound vertically at $3.0ms^{-1}$. Calculate the force on the roof if it has an area of $30m^2$ **An(36N)**
10. A hose with a nozzle 80mm in diameter ejects a horizontal stream of water at a rate of $0.044m^3s^{-1}$.
    (a) With what velocity will the water leave the nozzle
    (b) What will be the force exerted on a vertical wall situated close to the nozzle and at right-angle to the stream of water, if after hitting the wall;
        (i) The water falls vertically to the ground
        (ii) The water rebounds horizontally**An(8.75m/s, 385N, 770N)**
11. An astronaut is outside her space capsule in a region where the effect of gravity can be neglected. She uses a gas gun to move herself relative to the capsule. The gas gun fires gas from a muzzle of area $1.60mm^2$ at a speed of $150ms^{-1}$. The density of the gas is $0.800kgm^{-3}$ and the mass of the astronaut including her space suit is 130kg. calculate
    (a) The mass of gas leaving the gun per second
    (b) The acceleration of the astronaut due to gun, assuming that the change in mass is negligible **An($1.92x10^{-2}kgs^{-1}$, $2.22x10^{-2}ms^{-2}$)**
12. Sand is poured at a steady rate of $5.0gs^{-1}$ on to the pan of a direct reading balance calibrated in grams. If the sand falls from a height of 0.20m on to the pan and it does not bounce off the pan then, neglecting any motion of the pan, calculate the reading on the balance 10s after the sand first hits the pan. **An(0.051kg)**
13. A top class tennis player can serve the ball, of mass 57g at an initial horizontal speed of 50m/s. the ball remains in contact with the racket for 0.050s. calculate the average force exerted on the ball during the serve **An(57N)**
14. A motor car collides with a crash barrier when travelling at 100km/h and is brought to rest in 0.1s.
    (a) if the mass of the car and its occupants is 900kg calculate the average force on the car
    (b) Because of the seat belt, the movement of the driver whose mass is 80kg, is restricted to 0.20m relative tot the car. Calculate the average force exerted by the belt on the driver

An($2.5x10^5$N, $1.54x10^4$N)

15. A stone of mass 80kg is released at the top of a vertical cliff. After falling for by 3s, it reaches the foot of the cliff, and penetrates 9cm into the ground. What is;
    (a) The height of the cliff
    (b) The average force resisting penetration of the ground by the stone **An(45m, 400N)**
16. The blades of a large wind turbines, designed to generate electricity, sweeps pout an area of 1400m^2 and rotates about a horizontal axis which points directly into a wind of speed 15m/s



    (a) Calculate the mass of air passing per second through the area swept out by the blades (take the density of air to be 1.2 kg/m^3)
    (b) The mean speed of the on the far side of the blades is reduced to 13m/s. how much kinetic energy is lost by the air per second **An($2.5x10^4$kg/s, $7.1x10^5$J/s))**
17. A ball of mas $6.0x10^{-2}$kg moving at 15ms^{-1} hits a wall at right angles and bounces off along the same line at 10ms^{-1}
    (a) What is the magnitude of the impulse of the wall on the ball
    (b) The ball is estimated to be in contact with the wall for $3.0x10^{-2}$s, what is the average force on the ball **An(1.5Ns,50N)**
18. A body of mass 2.0kg and which is at rest is subjected to a force of 200N for 0.2s followed by a force of 400N for 0.30s acting in the same direction. Find
    (a) The total impulse on the body
    (b) The final speed of the body **An(160Ns,80ms^{-1})**

4.1.3: WHY LONG JUMPER BEND KNEES

By bending the knees, the time taken to come to rest is increased, which reduces the rate of change of momentum, therefore the force on the jumpers legs is reduced thus less pain on the legs.

Questions

1. Explain why, when catching a fast moving ball, the hands are drawn backwards while ball is being brought to rest.
2. Explain why a long jumper must land on sand
3. Why is it much more painful to be hit by a hailstone of mass 0.005kg falling at 5m/s which bounces off your head than by a raindrop of the same mass and falling at the same velocity but which breaks up on hitting you and does not bounce? (numerical answered is required)

4.1.4: LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that for a system of colliding bodies, their total linear momentum remains constant in a given direction provided no external forces acts on them.

Suppose a body A of mass m, and velocity U_1, collides with another body B of mass m_2 and velocity U_2 moving in the same direction

By principle of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Total momentum before collision | Total momentum after collision

4.1.5: Proof of the law of conservation of momentum using Newton's law

Let two bodies A and B with masses m_1 and m_2 moving with initial velocities u_1 and u_2 and let their velocities after collision be v_1 and v_2 respectively for time t with (v_1<v_2)

By Newton's 2^{nd} law:

Force on m_1: $F_1 = \frac{m_1(v_1-u_1)}{t}$

Force on m_2: $F_2 = = \frac{m_2(v_2-u_2)}{t}$

By Newton's 3rd law: $\boldsymbol{F_1 = -F_2}$

$\frac{m_1(v_1-u_1)}{t} = -\frac{m_2(v_2-u_2)}{t}$

$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$

$\therefore m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Hence $m_1u_1 + m_2u_2 =$ **constant**

4.1.6: COLLISIONS

In an isolated system, momentum is always conserved but this is not always true of the kinetic energy of the colliding bodies.

In many collisions, some of the kinetic energy is converted into other forms of energy such as heat, light and sound.

Types of collisions

1. **Elastic collisions**

   It is also perfectly elastic collision. This is a type of collision in which all kinetic energy is conserved. Eg collision between molecules, electrons.

2. **Inelastic collision**

   This is a type of collision in which the kinetic energy is not conserved.

3. **Completely inelastic collision**

   This is a type of collision in which the bodies stick together after impact and move with a common velocity. Eg a bullet embedded in a target

4. **Explosive collision (super elastic)**

   This is one where there is an increase in K.E.

Summary

Elastic collision	**Inelastic collision**	**Perfectly inelastic**
❖ Linear momentum is conserved	❖ Linear momentum is conserved	❖ Linear momentum is conserved
❖ Kinetic energy is conserved	❖ K.e is not conserved	❖ K.e is not conserved
❖ Bodies separate after collision	❖ Bodies separate after collision	❖ Bodies stick together and move with a common velocity
❖ Coefficient of restitution (elasticity)=1 (e=1)	❖ Coefficient of restitution is less than 1 (e<1)	❖ e=0

4.1.7: Mathematic treatment of elastic collision

Consider an object of mass m, moving to the right with velocity u_1. If the object makes a head-on elastic collision with another body of mass m_2 moving with a velocity u_2 in the same direction. Let v_1 and v_2 be the velocities of the two bodies after collision.

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$m_1(u_1-v_1) = m_2(v_2-u_2)$-------------------[1]

For elastic collision k.e is conserved

$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1{v_1}^2 + \frac{1}{2} m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ ---------------[2]

from equation 1 and 2 then

$$\frac{m_1(u_1-v_1)}{m_1(u_1^2-v_1^2)} = \frac{m_2(v_2-u_2)}{m_2(v_2^2-u_2^2)}$$

$$\frac{(u_1-v_1)}{(u_1+v_1)(u_1-v_1)} = \frac{(v_2-u_2)}{(v_2+u_2)(v_2-u_2)}$$

$$\frac{1}{(u_1+v_1)} = \frac{1}{(v_2+u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

$$v_2 - v_1 = -(u_2 - u_1)$$

Example

1. A particle P of mass m_1, travelling with a speed u_1 makes a head-on collision with a stationary particle Q of mass m_2. If the collision is elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Show that for $\beta = \frac{m_1}{m_2}$

(i) $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ (ii) $\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

Solution

By law of conservation of momentum

$m_1u_1 = m_1v_1 + m_2v_2$--------------[x]

$(u_1 - v_1) = \frac{m_2}{m_1}v_2$

Therefore $u_1 - v_1 = \frac{v_2}{\beta}$

$\beta(u_1 - v_1) = v_2$---------------------- [1]

for elastic collision k.e is conserved

½ $m_1u_1^2$ =½ $m_1{v_1}^2$ +½ $m_2{v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2\ )$

$\frac{m_1}{m_2}({u_1}^2 - {v_1}^2) = {v_2}^2$

$\beta(u_1^2 - v_1^2) = v_2^2$----------------------[2]

equating [1] and [2]

$\beta(u_1^2 - v_1^2) = [\beta(u_1 - v_1)]^2$

$\beta(u_1^2 - v_1^2) = \beta^2(u_1 - v_1)(u_1 - v_1)$

$(u_1 - v_1)(u_1 + v_1) = \beta(u_1 - v_1)(u_1 - v_1)$

$(u_1 + v_1) = \beta(u_1 - v_1)$

$v_1 + \beta v_1 = \beta u_1 - u_1$

$v_1(1+\beta) = u_1(\beta - 1)$

$\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$

ii) From $\frac{u_1}{v_1} = \frac{\beta+1}{\beta-1}$ ------------------ [xx]

from equation[1] : $v_2 = \beta(u_1 - v_1)$

$v_2 = \beta u_1 - \beta v_1$

$u_1 = \frac{v_2+\beta v_1}{\beta}$ put into (xx)

$$\frac{\left(\frac{v_2+\beta v_1}{\beta}\right)}{v_1} = \frac{(1+\beta)}{(\beta-1)}$$

$(v_2 + \beta v_1)(\beta - 1) = (1+\beta)\beta v_1$

$\beta v_2 + \beta^2 v_1 - v_2 - \beta v_1 = \beta v_1 + \beta^2 v_1$

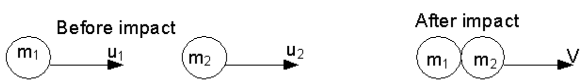
$\beta v_2 - v_2 = 2\beta v_1$

$v_2(\beta - 1) = 2\beta v_1$

$\frac{v_2}{v_1} = \frac{2\beta}{\beta-1}$

4.1.8: Mathematical treatment of perfectly inelastic collision

Suppose a body of mass m_1 moving with velocity u_1 to the right makes a perfectly inelastic collision with a body of mass m_2 moving with velocity u_2 in the same direction



By law of conservation

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Total kinetic energy before collision

k.e$_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

Total kinetic energy after collision

k.e$_f = \frac{1}{2}(m_1 + m_2)v^2$

Loss in k.e= k.e$_i$ – k.e$_f$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2$$

Numerical examples

1. Ball P, Q and R of masses m_1, m_2 and m_3 lie on a smooth horizontal surface in a straight line. The balls are initially at rest. Ball P is projected with a velocity u_1 towards Q and makes an elastic collision with Q. if Q makes a perfectly in elastic collision with R, show that R moves with a velocity.

$$v_2 = \frac{2\,m_1m_2u_1}{(m_1 + m_2)(m_2 + m_3)}$$

Solution

Elastic collision of P and Q:

Conservation of momentum:

$m_1u_1 = m_1v_P + m_2v_Q$

$v_P = u_1 - \frac{m_2v_Q}{m_1}$.......(1)

Conservation of kinetic energy:

$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_P^2 + \frac{1}{2}m_2v_Q^2$.....(2)

Putting [1] into [2

$m_1u_1^2 = m_1\left(u_1 - \frac{m_2v_Q}{m_1}\right)^2 + m_2v_Q^2$

$v_Q = \frac{2m_1u_1}{m_1+m_2}$......(3)

In elastic collision of Q and R:

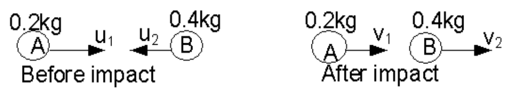
$m_2v_Q + m_30 = (m_2 + m_3)v_2$

$m_2\frac{2m_1u_1}{m_1+m_2} = (m_2 + m_3)v_2$

$v_2 = \frac{2m_1m_2u_1}{(m_1+m_2)(m_2+m_3)}$

2. A 0.2kg block moves to the right at a speed of $1ms^{-1}$ and meets a 0.4kg block moving to the left with a speed of $0.8ms^{-1}$. Find the find velocity of each block if the collision is elastic.

Solution



By law of conservation

$M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

$(0.2x1) + (0.4x - 0.8) = 0.2v_1 + 0.4v_2$

$0.2 - 0.32 = 0.2v_1 + 0.4v_2$

$v_1 + 2v_2 = -0.6$---------------------- [1]

for elastic collision K.E is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$0.2x1^2 + 0.4x(-0.8)^2 = 0.2v_1^2 + 0.4v_2^2$

$0.2 + 0.256 = 0.2v_1^2 + 0.4v_2^2$

$v_1^2 + 2v_2^2 = 2.28$ -----------------------[2]

But from [1] $v_1 = -0.6 - 2v_2$ put into (2)

$v_1^2 + 2v_2^2 = 2.28$

$2v_2^2 + (0.6 - 2v_2)^2 = 2.28$

$6v_2^2 + 2.4v_2 - 1.92 = 0$

v_2 = 0.4m/s , $v_2 = -0.8$m/s

v_2 = 0.4m/s is correct since m_2 is in front it supposed to move faster

Therefore from (1)

$v_1 + 2v_2 = -0.6$

$v_1 + 2x0.4 = -0.6$

$v_1 = -1.4m/s$

3. A truck of mass 1 tonne travelling at 4m/s collides with a truck of mass 2 tonnes moving at 3m/s in the same direction. If the collision is perfectly inelastic, calculate;

(i) Common velocity

(ii) Kinetic energy converted to other forms during collision

Solution

By law of conservation of momentum

$M_AU_A + M_BU_B = (M_A + M_B)V$

$(1000x4) + (2000x3) = (1000 + 2000)v$

$V = 3.3333ms^{-1}$

ii) Initial K.e $= \frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

$= ½\,x\,1000x\,4^2 + ½x2000x\,3^2 = 17000J$

Final k.e$_f$ $= \frac{1}{2}(M_A + M_B)V^2$

$= ½(1000 + 2000)(3.3333)^2$

$= 16666.67J$

Kinetic energy converted = k.e$_i$ – k.e$_f$

$= 17000 - 16666.67$

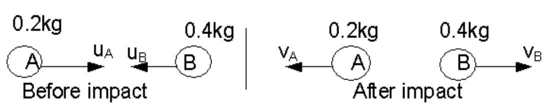
$= 333.33Joules$

4. Two particles of masses 0.2kg and 0.4kg are approaching each other with velocities $4ms^{-1}$ and $3ms^{-1}$ respectively. On collision, the first particle reverses, its direction and moves with a velocity of $2.5ms^{-1}$. find the;

(i) velocity of the second particle after collision

(ii) percentage loss in kinetic energy

Solution



By law of conservation of momentum

$M_AU_A + M_BU_B = M_AV_A + M_BV_B$

$0.2x4 + 0.4x - 3 = 0.2x2.5 + 0.4V_B$

$V_B = 0.25m/s$

The velocity of the second particle is 0.25m/s in opposite direction

ii) Initial k.e$_i$ $= \frac{1}{2}M_AU_A^2 + \frac{1}{2}M_BU_B^2$

$= ½\,(0.2x4^2 + 0.4x[-3]^2) = 3.4J$

Final K.e$_f$ $= \frac{1}{2}M_AV_A^2 + \frac{1}{2}M_BV_B^2$

$= \frac{1}{2}x\,0.2x2.5^2 + \frac{1}{2}x0.4x0.25^2 = 0.6475J$

Loss in kinetic energy = k.e$_i$ – k.e.$_f$

$= 3.4 - 0.6375 = 2.7625J$

% loss in k.e. $= \frac{loss\ of\ k.e}{k.e_i} x100\%$

$= \frac{2.7625}{3.4} x100\% = 81.25\%$

5. A bullet of mass $1.5 \ x\ 10^{-2} kg$ is fired from a rifle of mass $2.7x10^{2} kg$ with a muzzle velocity of 100km/h. Find the recoil velocity of the rifle.

Solution

$V_b = \frac{100\ x\ 1000}{60x60} = 27.78m/s$

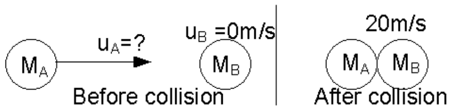
$M_g V_g = M_b V_b$

$3V_g = 1.5\ x\ 10^{-2} x27.78$

$V_g = 0.14m/s$

6. A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of 20m/s. Calculate
   (i) The speed with which the bullet hits the wood
   (ii) The kinetic energy lost

Solution



By the principle of conservation of momentum

$M_A U_A + M_B U_B = (M_A + M_B)V$

$(0.02xU_A) + (0.4x0) = (0.02 + 0.4)x20$

$u_A = 420m/s$

The original velocity of the bullet was ***420m/s***

Initial K.e $= \frac{1}{2} M_A U_A^2 + \frac{1}{2} M_B U_B^2$

$= \frac{1}{2} x\ 0.02x\ 420^2 + \frac{1}{2} x0.4x0^2 = 1764J$

Final K.e$_f$ $= \frac{1}{2}(M_A + M_B)V^2$

$= {}^1/_2 x(0.02 + 0.04)x(20)^2 = 84J$

Loss in kinetic energy = k.e$_i$ – k.e.$_f$

$= 1764 - 84 = 1680J$

7. A particle **P** of mass m_1 moving at a speed u_1 collides head on with a stationery particle Q of mass m_2. the collision is perfectly elastic and the speeds of P and Q after impact are v_1 and v_2 respectively. Given that $\alpha = \frac{m_2}{m_1}$
   (i) Determine the value of α if $u_1 = 20v_2$
   (ii) Show that the fraction of energy lost by P is $\frac{4\alpha}{(1+\alpha)^2}$

Solution

(i) $m_1 u_1 = m_1 v_1 + m_2 v_2$

$m_1(u_1 - v_1) = m_2 v_2$

$(u_1 - v_1) = \alpha v_2$……………………(1)

$v_1 = u_1 - \alpha v_2$……………………(2)

½ $m_1 u_1^2$ =½ $m_1 {v_1}^2$ +½ $m_2 {v_2}^2$

$m_1(u_1^2 - v_1^2) = m_2({v_2}^2)$

$(u_1^2 - v_1^2) = \alpha v_2^2$--------------------[3]

equating [3] ÷ [1] : $\frac{\alpha(u_1^2 - v_1^2)}{\alpha(u_1 - v_1)} = \frac{\alpha v_2^2}{\alpha v_2}$

$\frac{(u_1 - v_1)(u_1 + v_1)}{(u_1 - v_1)} = \frac{v_2^2}{v_2}$

$(u_1 + v_1) = v_2$ …………………………((4)

Put (2) into (4)

$(u_1 + u_1 - \alpha v_2) = v_2$

$2u_1 = (1 + \alpha)\ v_2$ …………………………((5)

but $u_1 = 20v_2$

$40v_2 = (1 + \alpha)\ v_2$

$\alpha = 39$

(iii) k.e of p before collision= ½ $m_1 u_1^2$

k.e of p after collision= ½ $m_1 v_1^2$

energy lost = ½$m_1 u_1^2$ – ½ $m_1 v_1^2$

fraction of energy lost $= \frac{½ m_1 u_1^2 - ½\ m_1 v_1^2}{½\ m_1 u_1^2}$

fraction of energy lost $= \frac{(u_1^2 - v_1^2)}{u_1^2} = \frac{(u_1 - v_1)(u_1 + v_1)}{u_1^2}$

from (i) above $(u_1 + v_1) = v_2$, $(u_1 - v_1) = \alpha v_2$

$u_1 = \frac{(1+\alpha)}{2} v_2$

fraction of energy lost $= \frac{(\alpha v_2)(v_2)}{\left[\frac{(1+\alpha)}{2} v_2\right]^2} = \frac{4\alpha}{(1+\alpha)^2}$

8. A body explodes and produces two fragments of masses m and M. If the velocities of the fragments are u and v respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{E_1}{E_2} = \frac{M}{m}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M

Solution

$$E_1 = \frac{1}{2}mu^2 \qquad and \qquad E_2 = \frac{1}{2}Mv^2$$

By law of conservation of linear momentum :

$$mu = -Mv$$

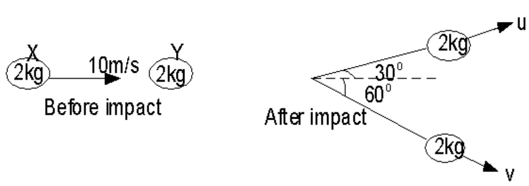
$$\therefore v = \frac{-mu}{M}$$

$$E_2 = \frac{1}{2}M\left(\frac{-mu}{M}\right)^2 = \frac{1}{2}\frac{m^2u^2}{M}$$

$$\frac{E_1}{E_2} = \frac{\left(\frac{1}{2}mu^2\right)}{\left(\frac{1}{2}\frac{m^2u^2}{M}\right)} = \frac{M}{m}$$

9. An object X of mass 2kg, moving with a velocity $10ms^{-1}$ collides with a stationary object Y of equal mass. After collision X moves with speed U at an angle of 30° to its initial direction while Y moves with a speed of Y at an angle of 90° to the new direction.
   (i) Calculate the speeds U and Y (05marks)
   (ii) Determine whether the collision is elastic or not. (03marks)

Solution



$(\rightarrow)$: $2x10 = 2ucos30 + 2vcos60$

$$20 = 2u\frac{\sqrt{3}}{2} + 2v\frac{1}{2}$$

$v = 20 - u\sqrt{3}$ ……………. [1]

$(\uparrow)$: $0 = 2usin30 - 2vsin60$

$$2usin30 = 2vsin60$$

$$\frac{u}{2} = v\frac{\sqrt{3}}{2}$$

$u = v\sqrt{3}$…………..[2]

Put into [1]: $v = 20 - \sqrt{3}\, v\sqrt{3}$

$$4v = 20$$

$v = 5ms^{-1}$

$u = v\sqrt{3}. = 5\sqrt{3} = 8.66ms^{-1}$

i. Total K.E before collision

K.e= $\frac{1}{2}x2x10^2 = 100J$

Total K.e after collision

$= 1/_2\, x2x(5)^2 + 1/2x2x(5\sqrt{3})^2 = 100J$

Since kinetic energy is conserved then the collision is elastic

Exercise:9

1. A 4kg ball moving at 8m/s collides with a stationery ball of mass 12kg, and they stick together. Calculate the final velocity and the kinetic energy lost in impact **An [2m/s, 96J]**
2. A body of mass 6kg moving at $8ms^{-1}$ collides with a stationary body of mass 10kg and sticks to it. Find the sped of the composite body immediately after impact **An(3m/s)**
3. A bullet of mass 6g is fired from a gun of mass 0.50kg. if the muzzle velocity of the bullet is $300ms^{-1}$, calculate the recoil velocity of the gun **An(3.6m/s)**
4. A body A of mass 4kg moves with a velocity of $2ms^{-1}$ and collides head on with another body, B of mass 3kg moving in the opposite direction at $5ms^{-1}$. After the collision the bodies move off together with v. Calculate v **An(-1m/s)**
5. A mass A of 6kg moving a velocity of 5m/s collides with a mass B of mass 8kg moving in the opposite direction at 3m/s.
   (a) calculate the final velocity if the masses stick together on impact
   (b) If the masses do not stick together but mass A continues along the same direction with a velocity of 0.5m/s after impact. Calculate the velocity of B. **An (0.43m/s, 0.38m/s)**
6. A sphere of mass 3kg moving with velocity 4m/s collides head–on with a stationary sphere of mass 2kg and imparts to it a velocity of 4.5m/s. calculate the;
   (i) velocity of the 3kg sphere after the collision.
   (ii) amount of energy lost by the moving bodies in the collision **An (1m/s, 2.25J)**
7. A 2kg object moving with a velocity of 8m/s collides with a 3kg object moving with a velocity $6ms^{-1}$ along the same direction. If the collision is completely inelastic, calculate the decrease in kinetic energy collision. **An [2.4J]**
8. Two bodies A and B of mass 2kg and 4kg moving with velocities of 8m/s and 5m/s respectively collide and move on in the same direction. Object A's new velocity is 6m/s.

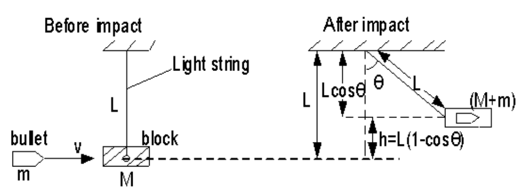
(i) Find the velocity of B after collision
(ii) Calculate the percentage loss in kinetic energy. **An(6m/s, 5.26%)**

9. A railway truck of mass $4x10^4$ kg moving at a velocity of 3m/s collides with another truck of mass $2x10^4$kg which is at rest. The coupling join and the trucks move off together
   (i) What fraction of the first trucks initial kinetic energy remains as kinetic energy of two trucks after collision **An** $[^2/_3]$
   (ii) Is energy conserved in a collision such as this, explain your answer
10. A particle of mass 2kg moving with speed $10ms^{-1}$ collides with a stationary particle of mass 7kg. Immediately after impact the particles move with the same speeds but in opposite directions. Find the loss in kinetic energy during collision. **An(28J)**
11. A 2kg object moving with a velocity of $6ms^{-1}$ collides with a stationary object of mass 1kg. If the collision is perfectly elastic, calculate the velocity of each object after collision. **An[$2ms^{-1}$, $8ms^{-1}$]**
12. A body of mass **m** makes a head on , perfectly elastic collision with a body of mass **M** initially at rest. Show that $\frac{\Delta E}{E_0} = \frac{4\left(\frac{M}{m}\right)}{\left(1+\frac{M}{m}\right)^2}$ where $E_0$is original kinetic energy of the mass **m** and ΔE the energy it loses in the collision
13. A metal sphere of mass m_1, moving at velocity u_1 collides with another sphere of mass $m_2$moving at velocity $u_2$in the same direction. After collision the spheres stick together and move off as one body. Show that the loss in kinetic energy E during collision is given by

$$E = \frac{\beta(u_1 + u_2)^2}{2(m_1 + m_2)} \quad where\ \beta = m_1 m_2$$

14. A stationary radioactive nucleus disintegrates into an α −particle of relative atomic mass 4, and a residual nucleus of relative atomic mass 144. If the kinetic energy of the α −particle is $3.24x10^{-13}$J, what is the kinetic energy of the residual nucleus **An($9x10^{-15}$J)**
15. On a linear air-track the gliders float on a cushion of air and move with negligible friction. One such glider of mass 0.50kg is at rest on a level track. A student fires an air rifle pellet of mass $1.5x10^{-3}$kg at the glider along the line of the track. The pellet embeds it's in the glider which recoil with a velocity of 0.33m/s. calculate the velocity ta which the pellet struck
An($1.1x10^2$m/s)

4.1.9: BALLISTIC PENDULUM



Resolving along the vertical gives $Lcos\theta$
But $L = Lcos\theta + h$
$h = L - Lcos\theta = L(1 - cos\theta)$
The device illustrates the laws of conservation of momentum and mechanical energy

a) **During impact**
- Mechanical energy is not conserved because of friction and other non conservative forces
- Linear momentum is conserved in the horizontal direction along which there is no external force
  If V_c is the velocity of combined mass just after collision
  $Mv + mx0 = (M + m)V_c$
  $mv = (m + M)V_c$……………………………………(i)
  The block was initially at rest.

b) **Swing after impact**
- Mechanical energy is conserved. The conserved gravitational force causes conversion of $k.e$ to $p.e$.
- Momentum is not conserved because an external resultant force (pull of the earth / weight) acts on the bullet-block system.

From (i) $k.e. = p.e.$

$$\frac{1}{2}(M+m)V_c^2 = (M+m)gh$$

$V_c^2 = 2gh$..........................(x)

But $h = L(1-cos\theta)$

$$V_c^2 = 2gL(1-cos\theta)$$

$V_C = \sqrt{2gL(1-cos\theta)}$ ....(2)

θ is the angle of swing

Factors on which angle of swing depends

- The speed of the bullet
- The length of the string

NB; the angle can be obtained from

$$h = L(1-cos\theta)$$
$$\frac{h}{L} = (1-cos\theta)$$
$$Cos\theta = \frac{L-h}{L}$$

$$\theta = Cos^{-1}\left(\frac{L-h}{L}\right)$$

OR: $V_C = \sqrt{2gL(1-cos\theta)}$

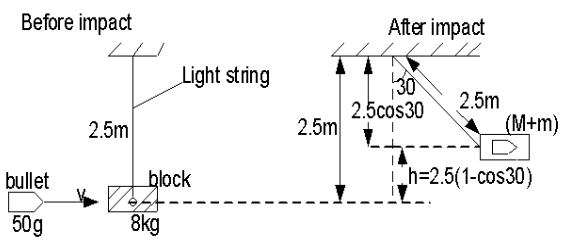
$$\frac{v_c^2}{2gL} = (1-cos\theta)$$

$$Cos\theta = \left(\frac{2gL - v_c^2}{2gL}\right)$$
$$\theta = cos^{-1}\left(\frac{2gL - v_c^2}{2gL}\right)$$

Examples

1. A bullet of mass 50g is fired horizontally into a block of wood of mass 8kg which is suspended by a string of length 2.5m. after collision the block swing upwards through an angle 30°. Calculate the velocity of the bullet assuming that it gets embedded in the block just after collision.

Solution



$$h = L(1-cos\theta) = 2.5(1-\cos 30) = 0.335m$$

Before impact (law of conservation of momentum)

$$mv + Mx0 = (M+m)V_C$$
$$\frac{50}{1000}v = \left(\frac{50}{1000}+8\right)V_C$$
$$0.05v = 8.05Vc$$
$$V_C = \frac{v}{161}$$

After impact (By conservation of mechanical energy)

$$\frac{1}{2}(m+M)V_c^2 = (m+M)gh$$
$$\frac{1}{2}(8+0.05)V_C^2 = (0.05+8)\,x9.81x0.335$$
$$V_c^2 = 6.5727$$
$$V_C = 2.564m/s$$

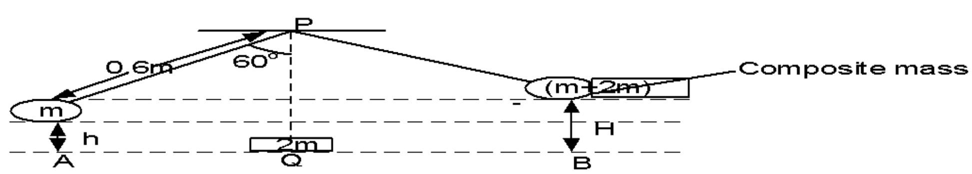
V_C is the velocity of bullet block system

But $V_C = \frac{v}{161}$

$$V = 161V_C = 161x2.564 = 412.804m/s$$

The velocity of the bullet is 412.804ms^{-1}

2. A steel ball of mass m is attached to an inelastic string of length 0.6m. The string is fixed to a point P so that the steel ball and the string can move in a vertical plane through P. The string is held out at an angle of 60° to the vertical and then released. At Q vertically below P, the wall makes a perfectly inelastic collision with the lump of plasticine of mass 2m so that the two bodies move together after collision



Calculate

(i) The velocity of the composite just after collision
(ii) The position of the composite mass with respect to point Q when the mass first comes to rest.
(iii) The composite mass now oscillates about the point Q, state two possible reasons why the composite mass finally comes to rest.

Solution

$h = L(1 - cos\theta) = 0.6(1 - cos60) = 0.3m$

Applying the law of conservation of energy at A

$P.E = K.E$

$mgh = \frac{1}{2}mv^2$

$V = \sqrt{2gh} = \sqrt{2x9.81x0.3} = 2.43m/s$

The velocity of mass m just before collision is 2.43m/s

Applying law of conservation of momentum at Q where collision occurs

i) $mv + 2mx0 = (m + 2m)Vc$

$2.43m = 3mV_C$

$V_c = 0.81ms^{-1}$

The velocity of the composite just after collision is $0.81ms^{-1}$

ii) Principle of mechanical energy at B

$K.E = P.E$

$\frac{1}{2}M_cV_c^2 = M_cgH$ but $M_c = (m + 2m)$

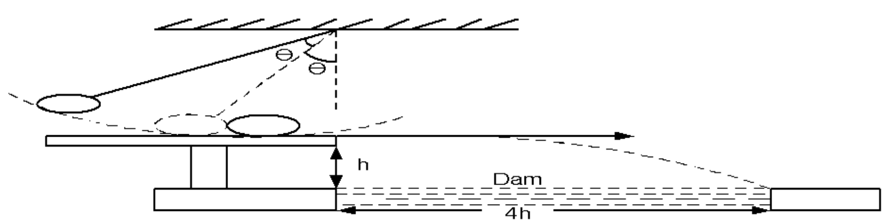
$H = \frac{1}{2}\frac{V_c^2}{g} = \frac{1}{2}x\frac{0.81^2}{9.81} = 0.33m$

iii) -Frictional force

-Air resistance

Exercise 10

1. A bullet of mass 40g is fired horizontally into freely suspended block of wood of mass 1.96kg attached at the end of an inelastic string of length 1.8m. given that the bullet gets embedded in the block and the string is deflected through an angle of 60° to the vertical . Find:
   (i) The initial velocity of the bullet **An[210m/s]**
   (ii) The maximum velocity of the block **An[42m/s]**
2. A bullet of mass 20g travelling horizontally at $100ms^{-1}$ embedded itself in the centre of a block of wood of mass 1kg which is suspended by a light vertical string 1m in length. Calculate the maximum inclination of the string to the vertical . **An(36.1°)**
3. A bullet of mass 50g travelling horizontally at $600ms^{-1}$ strikes a block of wood of mass 2kg which is suspended by a light vertical string so that its free to swing. The penetrates the block completely and emerges on the other side travelling at $400ms^{-1}$ in the same direction. As a result the block swings such that the string makes an angle of 25° with the horizontal. Calculate the lenght of the string. **An(1.719m)**
4. A block of wood of mass 1.00kg is suspended freely by a thread. A bullet of mass 10g is fired horizontally at the block and becomes embedded in it. The block swings to one suede rising a vertical distance of 50cm. with what speed did the bullet hit the block **An[319.4m/s]**
5. A circular ring is tied to a roof using a string of length, l and displaced such that it makes an angle of 2θ with the vertical, where $\theta = 30°$. It is then released to throw a speherical ball horizontally across the dam at a hegth h. It collides in elastically with the ball when at angle θ and move together until the ball leaves the bench horizontally to cross the dam of width 4h.



if the bench is frictionless and the masses are equal, show that $h = \frac{l(\sqrt{3}-1)}{32}$. Hence if $l = 128cm$ find the velociy with whoich the ball hits the ground

UNEB 2017 NO.1

(a) (i) State Newton's laws of motion (03marks)

(ii) A molecule of gas contained in a cube of side l strikes the wall of the cube repeatedly with a velocity u. Show that the average force F on the wall is given by $F = \frac{mu^2}{l}$ where m is the mass of the molecule (04marks)

(b) (i) Define the **linear momentum** and state the **law of conservation of linear momentum**. (02marks)

(ii) A body of mass m_1 moving with a velocity u, collides with another body of mass m_2 at rest. If they stick together after collision, find the common velocity with which they will move (04marks)

UNEB 2016 No 2

(a) (i) What is meant by **efficiency** of **a machine**. (01mark)

(ii) A car of mass $1.2x10^3kg.$ moves up an incline at a steady velocity of 15ms^{-1} agaisnt a frictional force of $6.0x10^3N$. The incline is such that the car rises 1.0m for every 10m along the incline. Calculate the out put power of the car engine. **An**($1.077x10^5W$) (04marks)

(b) (i) Define the **impulse** and **momentum**. (02marks)

(ii) An engine pumps water such that the velocity of the water leaving the nozzle is 15ms^{-1}. If the water jet is directed perpendicularly onto a wall and comes to a stop at the wall, calculate the pressure exerted on the wall **An**($2.25x10^5Nm^{-2}$) (04marks)

(c) (i) Define **inertia** (01mark)

(ii) Explain why a body placed on a rough plane will slide when the angle of inclination is increase.

(d) (i) State the conditions for a body to be in equilibrium under action of coplanar forces. (02marks)

(ii) Briefly explain the three states of equilibrium. (03marks)

UNEB 2013 No 3(a)

(I) State the law of conservation of linear momentum (01mark)

(II) A body explodes and produces two fragments of masses m and M. If the velocities of the fragments are u and v respectively, show that the ratio of kinetic energies of the fragments is

$$\frac{E_1}{E_2} = \frac{M}{m}$$

Where E_1 is the kinectic energy of m and E_2 is the kinectic energy of M (04marks)

UNEB 2011 NO.2

(a) State Newton's laws of motion (03marks)

(b) Use Newton's laws of motion to show that when two bodies collide their momentum is conserved (04marks)

(c) Two balls P and Q travelling in the same line in opposite directions with speeds of 6ms^{-1} and 15ms^{-1} respectively make a perfect inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the

(i) The velocity of P (04marks)

(ii) Change in kinetic energy **An[v=2.08ms^{-1},278.38J]** (04marks)

(d) (i) what is an impulse of a force (01marks)

(ii)Explain why a long jumper should normally land on sand. (04marks)

UNEB 2010 NO.1

(a) i) State the law of conservation of linear momentum (01mark)

ii)Use Newton's laws to derive the a(i) (04marks)

(b) Distinguish between elastic and inelastic collision (01mark)

(c) An object X of mass M, moving with a velocity 10ms^{-1} collides with a stationary object Y of equal mass. After collision X moves with speed U at an angle of 30° to its initial direction while Y moves with a speed of Y at an angle of 90° to the new direction.

(iii) Calculate the speeds U and Y **An**($v = 5\text{ms}^{-1}$ $u = 8.66\text{ms}^{-1}$) (05marks)

(iv) Determine whether the collision is elastic or not. $An(50mJ)$ (03marks)

UNEB 2009 NO.1

a) i) Define the term impulse (01mark)

ii) State Newton's laws of motion (03marks)

b) A bullet of mass 10g travelling horizontally at a speed of 100ms^{-1} strikes a block of wood of mass 900g suspended by a light vertical string and is embedded in the block which subsequently swings freely. Find the;

(i) Vertical height through which the block rises (04marks)

(ii) Kinetic energy lost by the bullet (03marks)

[Hint k.e. lost $=\frac{1}{2}m_b u_b^2 - \frac{1}{2}m_b V_C^2$]

Where V_C is velocity of combined system.

m_b –is mass of the bullet

u_b is initial velocity of the bullet **An(6.2×10^{-2}m , 49.99J)**

UNEB 2008 NO 4

a) State

(i) Newton's laws of motion (03 marks)

(ii) The principle of conservation of momentum (01 mark)

b) A body A of mass M_1 moves with velocity U_1 and collides head on elasticity with another body B of mass M_2 which is at rest. If the velocities of A and B are V_1 and V_2 respectively and given that x= $\frac{m_1}{m_2}$ Show that;

i) $\frac{u_1}{v_1} = \frac{x+1}{x-1}$ (04 marks)

ii) $\frac{v_2}{v_1} = \frac{2x}{x-1}$ (03 marks)

c) Distinguish between conservative and non conservative forces (02 marks)

d) A bullet of mass 40g is fired from a gun at 200ms^{-1} and hits a block of wood of mass 2kg which is suspended by a light vertical string 2m long. If the bullet gets embedded in the wooden block

(i) Calculate the maximum angle the string makes with the vertical (06 marks)

(ii) State factors on which the angle of swing depends **An (53.4^o)** (01 mark)

UNEB 2006 No 2(c)

(i) State the work – energy theorem (01 mark)

(ii) A bullet of mass 0.1kg moving horizontally with a speed of 420ms^{-1} strikes a block of mass 2.0kg at rest on a smooth table becomes embedded in it. Find the kinetic energy lost if they move together.
An[8400J] (04 marks)

UNEB 2005

C i) Define linear momentum (01 mark)

i) State the law of conservation of linear momentum (01 mark)

ii) Show that the law in c(ii) above follows Newton's law of motion (03 marks)

iii) Explain why, when catching a fast moving ball, hands are drawn back while the ball is being brought to rest. (02 marks)

d). A car of mass 1000kg travelling at uniform velocity of 20ms^{-1}, collides perfectly inelastically with a stationary car of mass 1500kg, calculate the loss in kinetic energy of the car as a result of collision
An[1.68×10^5J) (04 marks)

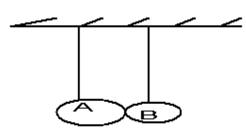
UNEB 2001 No 1

c)State the conditions under which the following will be conserved in a collision between two bodies.

i) Linear momentum [01mark]

ii) Kinetic energy [01mark]

d] Two pendula of equal length L have bobs A and B of masses 3m and m respectively the pendulum are lung with bobs in contact as shown.



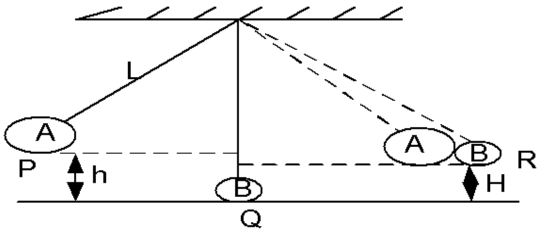
The bob A is displaced such hat the string makes an angle θ with the vertical and released. If A makes a perfectly inelastic collision with B, find the height to which B rises [08marks]

Solution

i) Linear momentum is conserved if there is no external resultant acting on the colliding bodies.

ii) Total kinetic energy is conserved if the collision is perfectly elastic i.e the bodies separate after collision

d]



At P: $h = L(1 - cos\theta)$

$P.e = K.e$ by conservation of energy

$3mgh = \frac{1}{2}3mv^2$

Where v is the velocity with which A is released

$3mgh = \frac{1}{2}3mv^2$

$gh = \frac{v^2}{2}$

$gL(1 - cos\theta) = \frac{v^2}{2}$

$v^2 = 2gL(1 - cos\theta)$

$v = \sqrt{2gL(1 - Cos\theta)}$ ----------------- [1]

At Q: Momentum is conserved

$3mv + mx0 = (3m + m)Vc$

Where Vc is the velocity of the combination

$3mv = 4mVc$

$3v = 4Vc$

$3\sqrt{2gL(1 - Cos\theta)} = 4Vc$

$Vc = \frac{3}{4}\sqrt{2gL(1 - Cos\theta)}$----------------[2]

At R : mechanical energy is conserved

$½ (3m + m)V_C^2 = (3m + m)gH$

H= $\frac{V_C^2}{2g} = \frac{\left(\frac{3}{4}\sqrt{2gL(1-Cos\theta)}\right)^2}{2g} = \frac{9gL(1-Cos\theta)}{16}$

B rises $\frac{9gL(1-Cos\theta)}{16}$

UNEB 2000 NO 1

a) i)State Newton's laws of motion [03marks]

ii) Define impulse and derive its relation to linear momentum of the body on which it acts. [03marks]

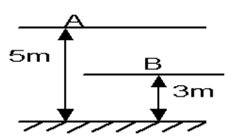
c) A ball of mass 0.5kg is allowed to drop from rest from a point at a distance of 5.0m above the horizontal concrete floor. When the ball first hits the floor, it rebounds to a height of 3.0m.

i) What is the speed of the ball just after the first collision with the floor [04marks]

ii) if the collision last 0.01s, find the average force which the floor exerts on the ball [05marks]

Solution

c)



i) By law of conservation of energy

k.e after collision =p.e at height of 3m

$½ mv^2 = mgh$

$v^2 = 2gh$

$v = \sqrt{2x9.81x3} = 7.67m/s$

Where v is the velocity with which it rebounds from the floor .

ii) Force = $\frac{change\ in\ momentun}{time}$

k.e on hitting floor =p.e at height of 5m

mgh = ½ mu^2

$u = \sqrt{2gh} = \sqrt{2x9.81x5}$ =**9.9ms^{-1}**

Since velocity is a vector quantity

$v = -7.67$ since it rebounds (moves in opposite direction)

$F = \frac{mv-mu}{t} = \frac{(0.5x9.9)-(0.5x-7.67)}{0.01} = 878.5N$

UNEB 1997 NO 2

a) Define the terms momentum [01marks]

b) A bullet of mass 300g travelling at a speed of 8ms^{-1} hits a body of mass 450g moving in the same direction as the bullet at 15ms^{-1}. The bullet and body move together after collision. Find the loss in kinetic energy [06marks]

c) i) State the work energy theorem [01mark]

ii)A ball of mass 500g travelling at a speed of 10ms^{-1} at 60° to the horizontal strikes a vertical wall and rebounds with the same speed at 120° from the original direction. If the ball is in contact with the wall for 8x10^{-3}, calculate the average force exerted by the ball.

Ans[625N] [06marks]

FORCE

Force is anything which changes a body's state of rest or uniform motion in a straight line

The unit of force is **a newton**

Definition: A Newton is a force which gives a body of mass 1kg an acceleration of $1ms^{-2}$

CONSERVATIVE AND NON CONSERVATIVE FORCES

1. **A conservative force** is a force for which the work done in moving a body around a closed path is zero.

Examples of conservative forces

- Gravitational force
- Electric force
- Elastic force
- Magnetic force

2. **A non-conservative force** is a force for which the work done in moving a body around a closed is not zero.

Examples of non- conservative force

- Frictional force
- Air resistance
- Viscous drag

Differences between conservative forces and non- conservative forces

Conservative forces	Non-conservative forces
Work done around a closed path is zero	Work done around a closed path is not zero
Work done to move a body from one point to another is independent on the path taken	Work done to move a body from one point to another is dependent on the path taken
Mechanical energy is conserved	Mechanical energy is not conserved

4.2.0: SOLID FRICTION

Friction is the force that opposes relative motion of two surfaces in contact.

4.2.1: Types of friction

1. Static friction

It's a force that opposes the tendency of a body to slide over another.

Limiting friction is the maximum frictional force between two surfaces in contact when relative motion is just starting.

2. Kinetic/sliding/dynamic friction

It's the force that opposes relative motion before two surfaces which are already in motion.

4.2.2: Laws of friction

1st law : Frictional forces between two surfaces in contact oppose their relative motion.

2nd law : Frictional forces are independent of the area of contact of the surfaces provided that normal reaction is constant.

3rd law : The limiting frictional force is directly proportional to the normal reaction but independent of relative velocity of surfaces.

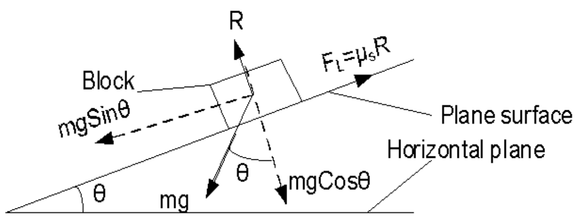
4.2.3: Molecular explanation for occurrence of friction

- Surfaces have very small projections and when placed together the actual area of contact of two surfaces is very small, hence the pressure at points of contact is very high. Projections merge to produce welding and the weldings have to be broken for relative motion to occur. This explains the fact that friction opposes relative motion between surfaces in contact

- When the area between the surfaces is changed, the actual area of contact remains constant. Therefore no change in friction. This explains the fact that friction is independent of the area of contact provided normal reaction is constant
- Increasing normal reaction, increases the pressure at the welds. This increases the actual area of contact to support the bigger load, and hence a greater limiting frictional force . Therefore friction is proportional to normal reaction.

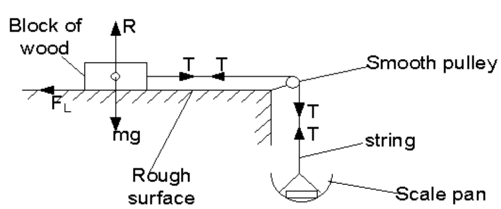
4.2.4: MEASUREMENT OF COEFFICIENT OF STATIC FRICTION

Method I



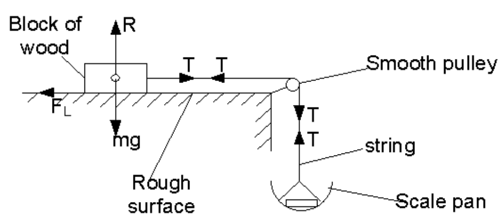
- Place a block on a horizontal plane.
- tilt the plane gently, until it **just begins** to slide.
- Measure and record the angle of tilt θ
- $\mu_s = tan\theta$
- Repeat the experiment with blocks of different masses
- Find the average value of μ_s

Method 2



- The mass m of the wooden block is determined and placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masses are added to the scale pan one at a time, till the block just slides
- The total mass M of the scale pan and the masses added is obtained.
- Coefficient of static friction $\mu = \frac{m}{M}$

4.2.5: Measurement of coefficient of kinetic friction

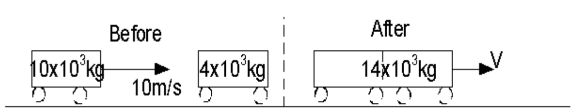


- The mass m of the wooden block is determined and placed on a horizontal plane surface.
- A string is attached to the block and passed over a smooth pulley carrying a scale pan at the other end.
- Small masses are added to the scale pan one at a time, till the block moves with a uniform speed
- The total mass M of the scale pan and the masses added is obtained.
- Coefficient of kinetic friction $\mu = \frac{m}{M}$

EXAMPLES

1. A truck of mass 10 tones moving at 10ms^{-1} draws into a stationary truck of mass 4 tones. They stick together and skid to a stop a long a horizontal surface. Calculate the distance through which the trucks skid, if the coefficient of kinetic friction is 0.25.

Solution



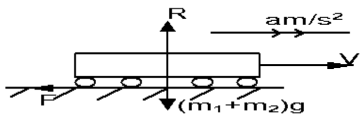
By law of conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$10^4x10 + (\ 4x10^3x0) \ = [10^4 + 4x10^3]v$$

$$v = 7.143ms^{-1}$$

When they skid to a s top, they experience a friction force



$F = \mu R$ but $R = (m_1 + m_2)g$

$F = \mu(m_1 + m_2) = 0.25(104 + 4x103)x9.81$

$Frictional\ force\ = 34335N$

Frictional force is the only resultant forces, therefore from Newton's 2nd law of motion

$34335 = (m_1 + m_2)A$

$34335 = (10^4 + 4x10^3)a$

$a \approx 2.453ms^{-2}$

The trucks come to a stop then

$a = -2.453ms^{-2}$(a deceleration)

To get the distance the trucks come to rest

u=7.143ms^{-1} v = 0m/s, a =2.453ms^{-2}

$v^2 = u^2 + 2as$

$0^2 = 7.143^2x2x - 2.453s$

$S = 10.4m$

Alternatively : Work done against friction=loss in k.e

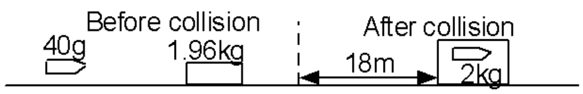
$$\mu(m_1 + m_2)gxs = \frac{1}{2}(m_1 + m_2)v^2$$

$$0.25x9.81xs = \frac{1}{2}x(7.143)^2$$

$$s = 10.4m$$

2. A 40g bullet strikes a 1.96 kg block of wooden placed on a horizontal surface just in front of the gun. The coefficient of kinetic friction between the block and the surface is 0.28. If the impact drives the block a distance of 18.0m before it comes to rest, what was the muzzle speed of the bullet

Solution



During impact: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$0.04u = 2v \ldots\ldots\ldots\ldots i$

$F = \mu R$ but $R = (m_1 + m_2)g$

$F = \mu(m_1 + m_2)g = 0.28(2)x9.81$

$Frictional\ force\ = 5.4936N$

Frictional force is the only resultant forces, therefore from $F = ma$

$5.4936 = (m_1 + m_2)a$

$5.4936 = (2)a$

$a \approx 2.7468ms^{-2}$

The trucks come to a stop then

$a = -2.7468ms^{-2}$ (a deceleration)

u=7.143ms^{-1} v = 0m/s, a =2.7468ms^{-2}

$v^2 = u^2 + 2as$

$0^2 = v^2 - 2.7468x18$

$v = 9.94ms^{-1}$

$0.04u = 2v$

$0.04u = 2x9.94$

$u = 497ms^{-1}$

Alternatively

Work done against friction=loss in k.e

$$\mu(m_1 + m_2)gxs = \frac{1}{2}(m_1 + m_2)v^2$$

$$0.28x9.81x18 = \frac{1}{2}x(v)^2$$

$$v = 9.94ms^{-1}$$

Put into $0.04u = 2v$

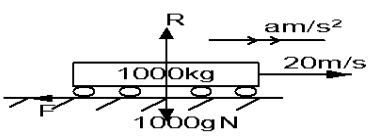
$0.04u = 2x9.94$

$u = 497ms^{-1}$

3. A car of mass 1000kg moving along a straight road with a speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 50m. Find the coefficient of kinetic friction between the tyres and the road.

Solution

$u = \frac{72x1000}{3600} = 20m/s$



$F = \mu R$ But $R = mg = 1000x9.81 = 9810N$

$F = 9810\,\mu$ --------- [1]

$F = ma$ --------------- [2]

To get the distance the car comes to rest

u=20m/s , v=0m/s, s=50m

$v^2 = u^2 + 2as$

$0 = 20^2 + 2ax50$

$a = -4ms^{-2}$

$F = 1000x - 4 = -4000N$

Frictional force =4000N

$F = 9810\,\mu$

$4000 = 9810\,\mu$

$\mu = 0.41$

Coefficient of friction = 0.41

Alternatively

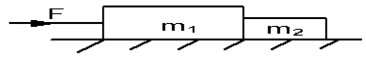
Work done against friction=loss in k.e

$$\mu(m)gxs = \frac{1}{2}(m)v^2$$

$$\mu x9.81x50 = \frac{1}{2}x(20)^2$$

$$\mu = 0.408$$

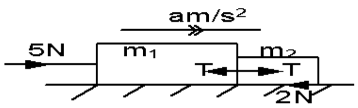
4. Two blocks of masses m_1=3kg and m_2=2kg are in contact on a horizontal table. A constant horizontal force F=5N is applied to the block of mass m_1 in the direction shown



There is a constant frictional force of 2N between the table and the block of mass m_2 but no frictional force between the table and the block of mass m_1. Find:

i) The acceleration of the two blocks
ii) The force of contact between the blocks

Solution



By Newton's 2nd law

For block m_1, $5 - T = 3a$ ---------- [1]

For block m_2 : $T - 2 = 2a$----------- [2]

Adding 1 and 2: $3 = 5a$

$a = 0.6ms^{-2}$

but from 2: $T - 2 = 2a$

$T = 2x0.6 + 2 = 3.2N$

Acceleration of two blocks =$0.6ms^{-2}$

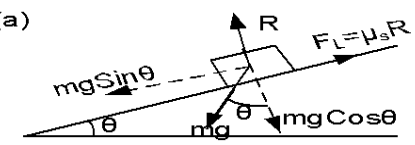
Force of contact= $3.2N$

5. A block of wood of mass 150g rests on an inclined plane. If the coefficient of static friction between the surface of contact is 0.3. find;

a) The greatest angle to which the plane may be tilted without the block slipping
b) The force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30°.

Solution

(a)

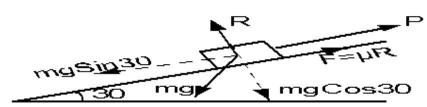


For the block not to slip then it experiences limiting friction

For limiting friction μ =tanθ

$\theta = tan^{-1}(\mu) = tan^{-1}(0.3) = 16.7°$

(b)



Using $F = ma$

$P + \mu R - mgsin30 = ma$

($a = 0$) no motion but $R = mgcos30$

$$P + 0.3x\frac{150}{1000}x9.81cos30 = \frac{150}{1000}x9.81sin30$$

P= $(\frac{150}{1000}x9.81sin30 - 0.3x\frac{150}{1000}x9.81cos30)$

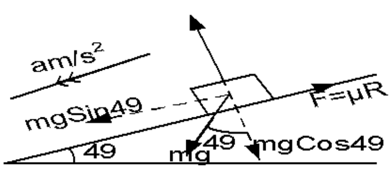
$P = 0.353N$

6. A car of mass 500kg moves from rest with the engine switched off down a road which is inclined at an angle 49° to the horizontal

a)Calculate the normal reaction

b)If the coefficient of friction between the tyres and surface of the road is 0.32. Find the acceleration of the car

Solution



a) $R = mgcos49$

R= $500x9.81cos49 = 3217.97N$

b) Using $F = ma$

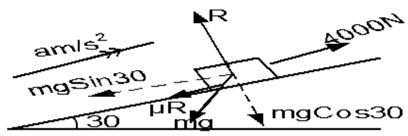
$mgsin49 - \mu R = ma$

$500x9.81sin49 - 0.32x3217.97 = 500a$

$a = 5.34ms^{-2}$

7. A car of mass 1000kg climbs a truck which is inclined at 30° to the horizontal. The speed of the car at the bottom of the incline is 36kmh⁻¹. If the coefficient of kinetic friction is 0.3 and engine exerts a force of 4000N how far up the incline does the car move in 5s?

Solution



u =36kmh^{-1} = $\frac{36x\,1000}{3600} = 10ms^{-1}$

Using $F = ma$

$4000 - (mgsin30 + \mu R) = ma$

$4000 - (1000x9.81sin30 + 0.3mgcos30) = 1000a$

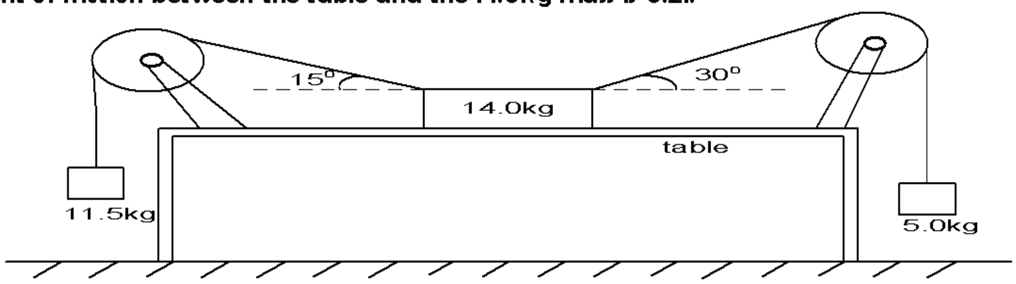
$a = -3.45\,ms^{-2}$

$S = ut + ½\,at^2$

$S = 10x5 + \frac{1}{2}x - 3.45x5^2$

$S = 6.9m$

8. The below shows three masses connected by inextensible strings which pass over smooth pulleys. The coefficient of friction between the table and the 14.0kg mass is 0.21.



If the system is released from rest, determine the

(i) Acceleration of the 14.0kg mass **An**(1.67ms^{-1})

(ii) Tension in each string **An(93.6N, 57.4N)**

Exercise:11

1. **A particle of weight 4.9N resting on a rough inclined plane of angle equal to tan^{-1}(5/12) is acted upon by a horizontal force of 8N. If the particle is on the point of moving up the plane, find coefficient of friction between the particle and the plane. An (μ = 0.72)**
2. **A box of mass 2kg rests on a rough inclined plane of angle 25°. The coefficient of friction between the box and the plane is 0.4.Find the least force applied parallel to the plane which would move the box up the plane. An[15.39N]**
3. **A particle of mass 0.5kg is released from rest and slides down a rough plane inclined at 30° to the horizontal. It takes 6 seconds to go 3 meter.**
   i. **Find the coefficient of friction between the particle and the plane**
   ii. **What minimum horizontal force is needed to prevent the particle from moving?**

   An[0.56, 0.086N]
4. **A parcel of mass 2kg is placed on a rough plane inclined at 45° to the horizontal, the coefficient of friction between the parcel and the plane is 0.25. Find the force that must be applied to the plane so that the parcel is just.**
   i. **Prevented from sliding down the plane**
   ii. **On the point of moving up the plane. An[10.39N, 17.32N]**

CHAPTER 5: WORK, ENERGY AND POWER

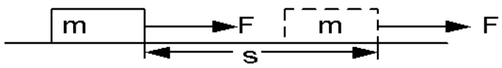
5.1.0: Work

5.1.1: Work done by a constant force

Work is said to be done when energy is transferred from one system to another

Case 1

When a block of mass m rests on a smooth horizontal



When a constant force F acts on the block and displaces it by x, then the work done by F is given by

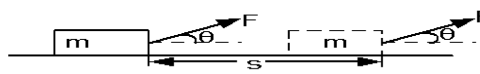
$$W = Fs$$

Definition

Work is defined as the product of force and distance moved in the direction of the force

Case II

If the force does not act in the direction in which motion occurs but at an angle to the it as shown below



$$W = (Fcos\theta)s$$

Definition

Work done is also defined as the product of the component of the force in the direction of motion and displacement in that direction

Note

1. Work done either can be positive or negative. If it is positive, then the force acts in the same direction of the displacement but negative if it acts oppositely.
   The work done by friction when it opposes one body sliding over it is negative.
2. Work and energy are scalar quantities and their S.I unit is Joules

Definition

A joule is the work done when a force of 1N causes a displacement of 1m in the direction of motion

Dimension of work

$W = Fs$

$[W] = [F][s]$

$= MLT^{-2}L$

[W] = ML^2T^{-2}

Explain why it is easier to walk on a straight road than an inclined road up hill.

When walking on a level ground, work is done only against the frictional force. While when walking up hill, work is done against both frictional force and the component of the weight of the person along the plane of the hill.

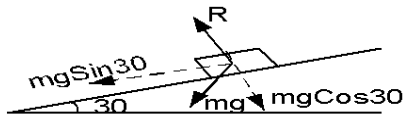
Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road

There is no net force on the bucket in the horizontal direction. The only force he exerts on the bucket is against the weight of the bucket and this force is perpendicular to the direction of motion. Therefore work done is $W = Fcos\theta = FCO90 = 0$. Hence the man does no work on the bucket

Examples

1. A block of mass 5kg is released from rest on a smooth plane inclined at an angle of 30° to the horizontal and slides through 10m. Find the work done by the gravitation force.

Solution



Work done by gravitational force

$W = mgsin30xd$

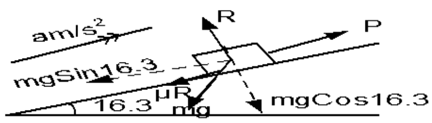
$W = 10x5x9.81sin30 = 245.25J$

2. A rough surface is inclined at $\tan^{-1}\left(\frac{7}{24}\right)$ to the horizontal. A body of mass 5kg lies on the surface and is pulled at a uniform speed a distance of 75cm up the surface by a force acting along a line of greatest slope. The coefficient of friction between the body and the surface is $\frac{5}{12}$. Find;

   a) Work done against gravity
   b) Work done against friction

Solution

$\theta = \tan^{-1}\left(\frac{7}{24}\right) = 16.3°$



a) Work done against friction

$W = \mu Rd \quad But \;\; R = mgcos\theta$

$W = \mu mgcos\theta d$

$$W = \frac{5}{12}x5x\frac{75}{100}x9.81cos16.3 = 14.71J$$

b) Work done against gravity

$W = mgsin\,\theta xd$

$$W = 5x9.81sin16.3x\frac{75}{100} = 10.35J$$

5.2.0 : ENERGY

This is the ability to do work.

When an interchange of energy occurs between two bodies, we can take the work done as measuring the quantity of energy transferred between them.

THE PRINCIPLE OF CONSERVATION OF ENERGY

It states that energy is neither created nor destroyed but changes from one form to another

5.2.1: KINETIC ENERGY

Kinetic energy is the energy possessed by a body due to its motion.

Formulae of kinetic energy

Consider a body of mass m accelerated from rest by a constant force, F so that in a distance, s it gains velocity, v

Then $v^2 = u^2 + 2as$ but $(u = 0)$

$a = \frac{v^2}{2s}$

$F = ma = \frac{mv^2}{2s}$

work done $= FxS = \frac{mv^2}{2s}s$

$w = \frac{mv^2}{2}$

by law of conservation of energy

$work\ done\ = k.e\ gained$

$$\boxed{k.e = \frac{1}{2}mv^2}$$

5.2.2: WORK-ENERGY THEOREM

It states that the work done by the net force acting on a body is equal to the change in its kinetic energy.

WORK-ENERGY THEOREM FORMULAR

Consider a body of mass m accelerated from u by a constant force F so that in a distance s it gains velocity v

$v^2 = u^2 + 2as$

$a = \frac{v^2-u^2}{2s}$ ---------------------- [1]

resultant $force\ F = ma = \frac{m(v^2-u^2)}{2s}$

But work done $= Fxs = \frac{m(v^2-u^2)}{2s}s$

$W = \frac{m(v^2-u^2)}{2}$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

This is the work-energy theorem.

Examples

1. A car mass 1000kg moving at 50ms^{-1} skid to rest in 4s under a constant retardation. Calculate the magnitude of the work done by the force of friction

Solution

a) Using $v = u + at$

$0 = 50 + 4a$

$a = -12.5 m/s^2$

Frictional force $= ma$

$= 1000x - 12. = 12500N$

$S = ut + ½\, at^2$

$S = 50x4 + ½\, x - 12.5x4^2$

$S = 100m$

W$= FxS = 12500x100$

Work done $= 1.25x10^6 J$

Alternatively

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W$= \frac{1}{2}x1000x50^2 - \frac{1}{2}\, x1000x0^2$

Work done $= 1.25x10^6 J$

2. A bullet travelling at 150ms^{-1} will penetrate 8cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4cm of the block.

Solution

Loss in k.e energy=work done against resistance

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = w$

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FxS$

$\frac{1}{2}mx0^2 - \frac{1}{2}mx150^2 = maxs$

$-\frac{1}{2}mx150^2 = max\frac{8}{100}$

$a = -140625$ms^{-2}

Using $v^2 = u^2 + 2as$

$v^2 = 150^2 + 2x(-140625)x\frac{4}{100}$

$v = 106.06ms^{-1}$

3. A constant force pushes a mass of 4kg in a straight line across a smooth horizontal surface. The body passes through a point A with a speed of 5m/s and then through a point B with a speed of 8m/s. B is 6m form A. Find the magnitude of the force acting on the mass.

Solution

$v^2 = u^2 + 2as$

$a = \frac{8^2 - 5^2}{2x6} = 3.25ms^{-2}$

$F = ma = 4x3.25 = 13N$

OR $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$Fx6 = \frac{1}{2}x4x(8^2 - 5^2)$

$F = 13N$

4. A body of mass 5kg slides over a rough horizontal surface. In sliding 5m, the speed of the body decrease from 8m/s to 6m/s, find

(i) Frictional force

(j) Coefficient of friction

Solution

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$Fx5 = \frac{1}{2}x5x(8^2 - 6^2)$

$F = 14N$

$F = \mu R$

$\mu = \frac{14}{5x9.81} = 0.286$

Alternatively $v^2 = u^2 + 2as$

$a = \frac{6^2 - 8^2}{2x5} = -2.8ms^{-2}$

$F = ma = 5x2.8 = 14N$

5. A bullet of mass 15g is fired towards a fixed wooden block and enters the block when travelling horizontally at 400m/s. It comes to rest after penetrating a distance of 25cm. find the

(i) work done against resistance of the wood

(ii) Magnitude of the resistance

Solution

(i) $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$W = \frac{1}{2}x0.015x(400^2 - 0^2) = 1200J$

(ii) $W = FxS$

$1200 = Fx0.25$

$F = 4800N$

6. A particle of mass 2kg is released from rest and falls freely under gravity. Find its speed when it has fallen a distance of 10m

Solution

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$2x9.8x10 = \frac{1}{2}x2x(v^2 - 0^2)$

$v = 14m/s$

7. A particle of mass 5kg falls vertically against a constant resistance. The particle passes through two points A and B 2.5m apart with A above B. Its speed is 2m/s when passing through A and 6m/s when passing. through B. Find the magnitude of the resistance

Solution

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

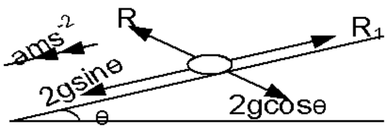
$$(5g - R)x2.5 = \frac{1}{2}x5x(6^2 - 2^2)$$

$$R = 17N$$

Incline planes

1. A rough slope of length 5m is inclined at angle of 30° to the horizontal. A body of mas 2kg is released from rest at the top of the slope and travels down the slope against a constant resistance. The body reaches the bottom of the slope with speed of 2m/s, find the magnitude of the resistance

Solution



$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$(2gsin\theta - R)x5 = \frac{1}{2}x2x(2^2 - 0^2)$$

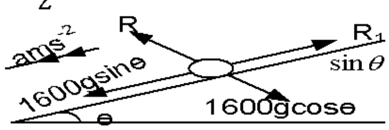
$$R = 9N$$

2. A car of mass 1600kg slides down a hill of slope 1 in 25. When the car descends 200m along at the hill, its speed increases from $3ms^{-1}$ to $10ms^{-1}$. Calculate
   (i) The change in the total kinetic energy
   (ii) Average value of resistance to motion

Solution

(i) $\Delta k.e = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2}x1600(10^2 - 3^2) = 72{,}800J$$



$$v^2 = u^2 + 2as$$

$$a = \frac{10^2 - 3^2}{2x200} = 0.228ms^{-2}$$

$$using\ \ F = ma$$

$$1600g\ sin\theta - R_1 = 1600a$$

$$R_1 = 1600x9.8x\frac{1}{25} - 1600x0.228 = 262.4N$$

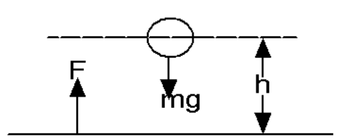
OR $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$(1600gsin\theta - R)x200 = \frac{1}{2}x1600(10^2 - 3^2)$$

$$R = 263.2N$$

5.2.3: GRAVITATIONAL POTENTIAL ENERGY

Potential energy is the energy that a body has due to its position in a gravitational field. Consider a body of mass m on the surface of the earth moved up a height h by a greater Force F.



Work done = $Fx\ S$

work done $= mgxh$

But work done =P.E gained at maximum height

$$\boxed{P.E = mgh}$$

Note

When a body is moving vertically upwards, it losses K.E but gains P.E and when moving downwards, it losses P.E and gains K.E

Definition

Elastic potential energy is energy possessed by a stretched or compressed elastic material eg spring.

P.E (elastic)= ½ ke^2

Where k is the spring constant and e is the compression /extension

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

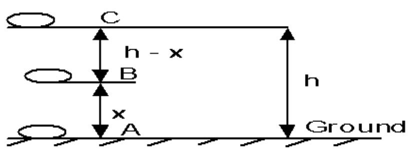
States that in a mechanical system the total mechanical energy is a constant provided that no dissipative forces act on the system.

Examples of dissipative forces are;
Frictional force, air resistance, viscous drag

Examples of principle of conservation of M.E

i) A body thrown vertically upwards

Consider a body of mass m projected vertically upwards with speed u from a point on the ground. Suppose that it has a velocity v at a point B at a height h above the ground provided no dissipative forces act.



At point A

$P.E = 0$ and K.E $= \frac{1}{2} mu^2$

Total energy $= K.E + P.E = \frac{1}{2} mu^2$

At point B

K.E $= \frac{1}{2} mv^2$ and $P.E = mgx$

But $v^2 = u^2 - 2gx$

Total energy $= \frac{1}{2} m(u^2 - 2gh) + mgx = \frac{1}{2} mu^2$

At point C

K.E $= \frac{1}{2} mv^2$ and P.E $= mgh$

$but\ v^2 = u^2 - 2gh$

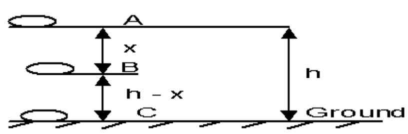
Total energy $= mgh + \frac{1}{2} m(u^2 - 2gh)$

$= \frac{1}{2} mu^2$

Since the total mechanical energy at all points is constant then the mechanical energy of a an object projected vertically upwards is conserved provided there is no dissipative force.

ii) A body falling freely from a height above the ground

Consider a body of mass 'm' at a height 'h' from the ground surface and at rest



At point A

$K.E = 0$ (at rest) and P.E $= mgh$

Total energy $= K.E + P.E = mgh$

At point B

K.E $= \frac{1}{2} mv^2$ and $P.E = mg(h - x)$

$but\ v^2 = 2gx$

Total energy $= \frac{1}{2} m\, 2gx + mg(h - x) = mgh$

At point C (just before impact)

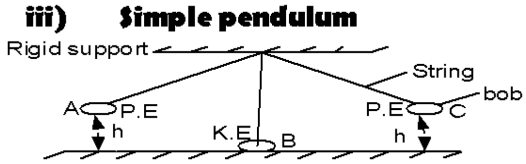
K.E $= \frac{1}{2} mv^2$ and $P.E = 0$ (ground level)

$v^2 = 2gh$

Total energy $= mgh$

Since the total mechanical energy at all points is constant then the mechanical energy of a freely falling object is conserved provided there is no dissipative force.

iii) Simple pendulum

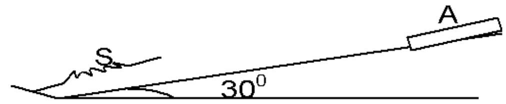


It consists of a bob that oscillates about equilibrium position B.

- At extreme ends A and C of the swing, the energy is potential energy and maximum since h is maximum.
- When passing through rest position B, the energy is kinetic energy and maximum; since the velocity at B is maximum and h = 0.
- At intermediate positions (i.e. between AB and BC) the energy is partly kinetic and partly potential.

Example

1. A block of mass 1kg is released from rest and travels down a rough incline of 30° to the horizontal a distance of 2m before striking a spring of force constant $100Nm^{-1}$. The coefficient of friction between the block and the plane is 0.1

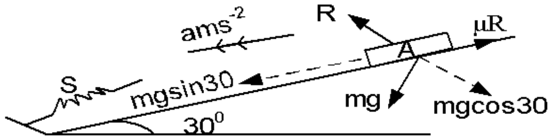


Calculate the:

(i) velocity of B just before it strikes the spring

(ii) maximum compression of the spring

solution



$$\boldsymbol{F = ma}$$

$ma = mgsin30 - \mu R$ **but** $R = mgcos30$

$$ma = mgsin30 - 0.1mgcos30$$

$$a = 4.055ms^{-2}$$

$$v^2 = u^2 + 2as$$

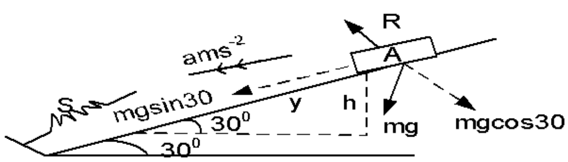
$$v = \sqrt{0^2 + 2x4.055x2} = 4.027ms^{-1}$$

(ii) $\frac{1}{2}ke^2 = \frac{1}{2}mv^2$

$$e = \sqrt{\frac{1x(4.027)^2}{100}} = 0.4027m$$

2. **A block of mass 0.2kg is released from rest and travels down a rough incline of 30° to the horizontal. The block compresses a spring of force constant $20Nm^{-1}$placed at the bottom of the plane by 10cm before its brought to rest. Find the distance the block travels down the incline before it comes to rest and its speed just before it reaches the spring**

solution



by conservation of energy

$\frac{1}{2}ke^2 = mgh$

but h=ysin30

$$\frac{1}{2}x20x0.1^2 = 0.2x9.81ysin30$$

$$y = 0.1m$$

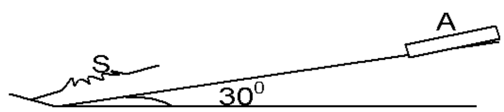
$$\frac{1}{2}ke^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}x20x0.1^2 = \frac{1}{2}mv^2$$

$$0.1 = \frac{1}{2}x0.1v^2$$

$$v = 1ms^{-1}$$

3. **An ideal mass less spring is compressed 3.0cm by a force of 100N. the same spring is placed at the bottom of a frictionless inclined plane which make an angle of 30 with the horizontal as shown below**



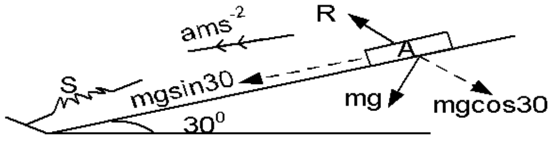
A 4.0kg mas released from rest at top of the incline and is brought to rest after compressing the spring 5.0cm. Find:

(I) The speed of the mass just before it reaches the spring

(II)The distance through which the mass slides before it reaches the spring

(III) The time taken by the mass to reach the spring

Solution



$\frac{1}{2}ke^2 = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{3330x(5x10^{-2})^2}{4}} = 1.44ms^{-1}$$

(ii) $\boldsymbol{F = ma}$

$$ma = mgsin30$$

$$a = 9.81sin30 = 4.9ms^{-2}$$

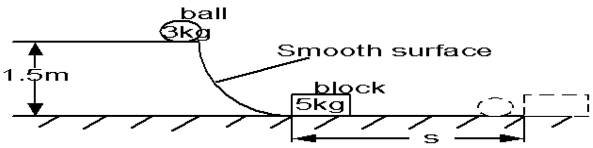
$$v^2 = u^2 + 2as$$

$$s = \frac{1.44^2 - 0^2}{2x4.9} = 0.21m$$

(iii) $v = u + at$

$$t = \frac{1.44 - 0}{4.9} = 0.294m$$

4. A ball of mas 3kg slides down a friction less surface and then strikes a stationary 5kg block on a horizontal surface as shown below



The coefficient of kinetic friction between the block and the table is 0.1. if the ball and the block stick together, how far do they slide before coming to rest

Solution

Before collision By conservation of energy:

$\frac{1}{2}mu^2 = mgh$

$$u^2 = 2x9.81x1.5$$

$$u = 5.42m/s$$

During collision: $m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$

$$3x5.42 = 8v$$

$$v = 2.03ms^{-1}$$

After collision:

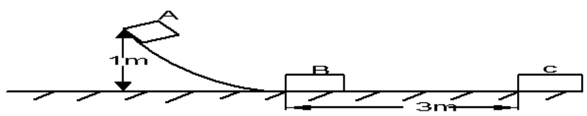
Work done against friction=loss in k.e

$$\mu(m_1 + m_2)gxs = \frac{1}{2}(m_1 + m_2)v^2$$

$$0.1x9.81xs = \frac{1}{2}x(2.03)^2$$

$$s = 2.1m$$

5. A ball of mas 2kg is released from rest at point A on a friction less track which is one quadrant of circles of radius 1m as shown below.



The block reaches point B with a velocity of 4m/s. From point B, it then slides on a level road to point C where it comes to rest.

(i) find the coefficient of sliding friction on the horizontal surface

(ii) how much work was done against friction as the body slides down from A to B

Solution

From B to C:

Work done against friction=loss in k.e

$$\mu(m)gxs = \frac{1}{2}(m)v^2$$

$$\mu x9.81x3 = \frac{1}{2}x(4)^2$$

$$\mu = 0.272$$

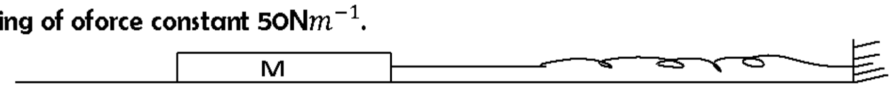
From A to B:

Work done against friction=change in M.E

$$= mgh - \frac{1}{2}(m)v^2$$

$$= 2x9.81x1 - \frac{1}{2}x2x(4)^2 = 3.62J$$

6. The figure below hows a wooden block M of mass 990g resting on a rough horizontal surface and attached to a spring of oforce constant 50Nm^{-1}.



When a sharp nail of mass 10g shot at close range ito the block, th spring is compressed by a distance of 20cm. If th work done against friction is $9x10^{-2}J$, Find the intial speed of the nail just before collision with the block.

Solution

After collision By conservation of energy:

K.e of the naill and block=incresae in P.E+Work again friction

$$\frac{1}{2}(m + M)\,v^2 = \frac{1}{2}kx^2 + 9x10^{-2}J$$

$$\frac{1}{2}(0.01 + 0.99)\,v^2 = \left(\frac{1}{2}x50x0.02^2 + 9x10^{-2}J\right)$$

$$v = 0.0141m/s$$

Befefore collision: $m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$

$$(0.01u) + 0.99x0 = (0.01 + 0.99)x0.0141$$
$$u = 1.41m/s$$

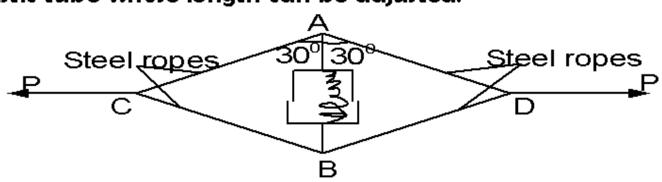
7. **A car of mas 1000kg increases its speed from 10ms^{-1} to 20ms^{-1} while moving 500m up a road inclined at an angle θ to the horizontal where sin $\theta = \frac{1}{20}$. There is a constant resistance to motion of 300N. Find the driving force exerted by the engine, assuming that it is constant**

 Solution

 Work done by engine=incresae in P.E+incresae in K.E+Work against resistance

$$F_D\mathbf{x500} = mghsin\,\theta + \left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right) + Fxs$$
$$F_D\mathbf{x500} = 1000x9.81x500x\frac{1}{20} + \frac{1}{2}x1000(20^2 - 10^2) + 300x500$$
$$F_D = 1100N$$

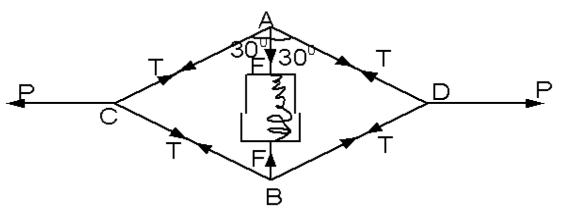
8. **A muscle exerciser consists of two steel ropes attached to the ends of a strong spring of force constant 500N/m contained in a plastic tube whose length can be adjusted.**



The spring has an uncompressed length of 0.80m. The ropes are pulled with equal and opposite forces, P so that the string is compressed to a length of 0.60m and the ropes make an angle of 30° with the length of the springs. Calculate;

 (i) Tension in each rope **(ii) Force p**

Solution



$$e = 0.8 - 0.6 = 0.2m$$
$$F = Ke = 2Tcos30°$$
$$T = \frac{ke}{2cos30°} = \frac{500x0.2}{2cos30°} = 57.7N$$
$$(\rightarrow): P = 2Tsin30°$$
$$P = 2x57.7x0.5 = 57.7N$$

Exercises12

1. **A car of mass 800kg and moving at 30ms^{-1} along a horizontal road is brought to rest by a constant retarding force of 5000N. calculate the distance the car moves while coming to rest. An(72m)**
2. A car of mass 1200kg moves 300m up a road which is inclined at an angle θ to the horizontal where sin $\theta = \frac{1}{15}$. By how much does the gravitational potential energy of the car increase **An(2.4x10^{5}J)**
3. **A car of mass 800kg moving at 20ms^{-1} is brought to rest by the application of brakes in a distance of 100m. Calculate the work done by the brakes and the force they exert assuming that it is constant an d that there is no other resistance to motion An(1.6x10^{5}J, 1.6x10^{3}N)**
4. **The speed of a dog-sleigh of mass 80kg and moving along horizontal ground is increased form 3.0ms^{-1} to 9.0ms^{-1} over a distance of 90m. find;**
 (a) The increase in the k.e of the sleigh
 (b) The force exerted on the sleigh by the dogs assuming that it is constant and there is no resistance to motion An(2.9x10^{3}J, 32N)
5. **A simple pendulum consisting of a small heavy bob attached to a light string of length of length 40cm is released from rest with the string at 60° to the downward vertical. Find the speed of the pendulum bob as it passes through its lowest point An(2.0ms^{-1})**

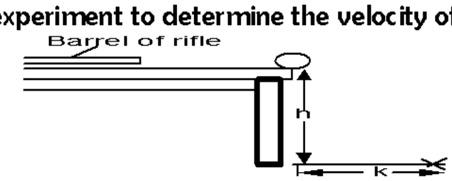
6. A car of mass 900kg accelerates from rest to a speed of $20ms^{-1}$ while moving 80m along a horizontal road. Find the tractive force exerted by the engine, assuming that it is constant and that there is a constant resistance to motion of 250N **An($2.5x10^3$N)**

7. A child of mass 20kg starts from rest at the top of a playground slide and reaches the bottom with a speed of $5.0ms^{-1}$. The slide is 5.0m long and there is a difference in height of 1.6m between the top and the bottom. Find
   (i) The work done against friction
   (ii) The average frictional force **An(70J, 14N)**

9. Two particles of masses 6.0kg and 2.0kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest with the string taut. Find the speed of the particles when the heavier one has descended 2.0m **An($4.5ms^{-1}$)**

10. A ball of mass 50g falls from a height of 2.0m and rebounds to a height of 1.2m. How much kinetic energy is lost on impact **An(0.4J)**

11. A student devises the following experiment to determine the velocity of a pellet from an air rifle



A piece of plasticine of mass **M** is balanced on the edge of a table such that it just fails to fall off. A pellet of mass, **m** is fired horizontally into the plasticine and remains embedded in it. As a result the plasticine reaches the floor a horizontal distance k away. The height of the table is **h**

   (i) show that the horizontal velocity of the plasticine with pellet embedded is $k\left(\frac{g}{2h}\right)^{1/2}$
   (ii) obtain an expression for the velocity of the pellet before impact with the plasticine

12. A model railway truck P, of mass 0.20kg and a second truck, Q o of mass 0.10kg are at rest on two horizontal straight rails, along which they can move with negligible friction. P is acted on by a horizontal force of 0.10N which makes an angle of 30^0 with the track. After P has travelled 0.50m, the force is removed and P then collides and sticks to Q. calculate;
   (a) The work done by the force
   (b) The speed of P before the collision
   (c) The speed of the combined trucks after collision **An**($4.3x10^{-3}$J, 0.66m/s, 0.44m/s)

13. A particle A of mass 2kg and a particle B of mass 1kg are connected by a light elastic string C and initially held at rest 0.9m apart on a smooth horizontal table with the string in tension. They are simultaneously released. The string releases 12J of energy as it contracts to its natural length.

Calculate the velocity acquired by each of the particles and find where the particles collide
An(2m/s, 4m/s, 0.3m from A)

14. A particle of mass 3kg and a particle Q of mass 1kg are connected by a light elastic string and initially held at rest on a smooth horizontal table with the string in tension. They are simultaneously released. The string releases 24J of energy as it contracts to its natural length. Calculate the velocity acquired by each of the particles . **An(2m/s, 4m/s, 0.3m from A)**

15. A bullet of mass $2.0x10^{-3}$kg is fired horizontally into a free- standing block of wood of mass $4.98x10^{-1}$kg, which it knocks forward with an initial speed of 1.2m/s
   (a) Estimate the speed of the bullet
   (b) How much kinetic energy is lost in the impact **An(300m/s, 89.64J)**
   (c) What becomes of the lost kinetic energy

16.

As shown in the diagram, two trolleys P and Q of mass 0.50kg and 0.30kg respectively are held together on a horizontal track against a spring which is in a state of compression.

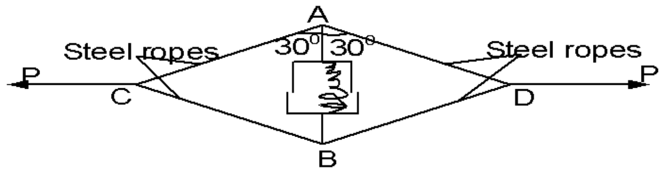
(a) When the spring is released the trolley separate freely and P moves to the left with an initial velocity of 6m/s. calculate

(i) Initial velocity of Q

(ii) The initial total kinetic energy of the system

(b) Calculate the initial velocity of Q if trolley P is held still when the spring under the same compression as before us released **An(10m/s, 24J, 12.5m/s)**

17. A muscle exerciser consists of two steel ropes attached to the ends of a strong spring contained in a telescopic tube. When the ropes are pulled sideways in opposite directions in the diagram below



The spring has an uncompressed length of 0.8m. the force F in newton required to compress the spring to a length x in meters is given by $F = 500(0.80 - x)$

The ropes are pulled with equal and opposite forces, P so that the string is compressed to a length of 0.60m and the ropes make an angle of 30° with the length of the springs

(a) Calculate the force F

(b) the work done in compressing the spring

(i) by considering forces at A or B, calculate the tension in each rope

(ii) by considering forces at C or D, calculate the force P **An(100N, 10J, 57.7N, 57.7N)**

5.3.0: POWER

It's the rate of doing work.

Its units are watts(W) or joule per second [Js^{-1}]

Power = $\frac{\text{Work done}}{\text{time}}$

P= $\frac{F\,x\,d}{t}$

P= $Fx\ \frac{d}{t}$

$P = Fxv$

Dimensions of power

$[P] = [F]x[v]$

[P] $= MLT^{-2}LT^{-1}$

[P] $= ML^2T^{-3}$

Numerical examples

1. A ball of mass of 0.1kg is thrown vertically up wards with an initial speed of 20m^{-1}. Calculate

i) the time taken to return to the thrower

ii) the maximum height

iii) the kinetic and potential energy of the ball half way up.

Solution

i) Using $v = u + gt$

$0 = 20 - 9.81t$

$t = 2.04s$

Time to return to the thrower= $2x2.04$

T= $4.08s$

ii) max height (v=om/s)

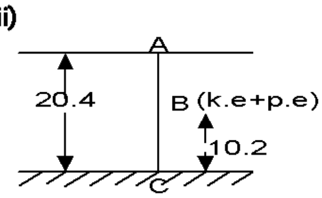
$v^2 = u^2 - 2gs_{max}$

$0 = 20^2 - 2x9.81s_{max}$

$s_{max} = \frac{400}{2x9.81}$

$s_{max} = 20.39m$

iii)



$k.e\ = \frac{1}{2}mv^2$ --------- (1)

But $v^2 = u^2 + 2gs$

$v^2 = 20^2 + 2x9.81x10.2$

$v = 14.14m/s$

$k.e = \frac{1}{2}x0.1x14.14^2$

$k.e\ = 9.96J$

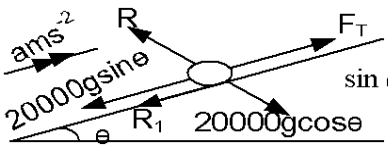
$p.e = mgh$

$p.e = 0.1x9.81x10.2$

$p.e = 10.01J$

2. A train of mass 20000kg moves at a constant speed of 72kmh^{-1} up a straight incline against a frictional force of 128. The incline is such that the train rises vertically one meter for every 100m travelled along the incline. Calculates the necessary power developed by the train.

Solution



$\sin\theta = \frac{1}{100}$

Using $F = ma$

$F_T - (mgSin\theta + R_1) = ma$

$a = 0ms^{-2}$ constant speed

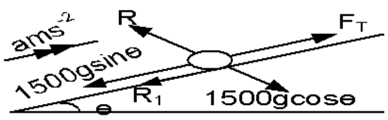
$$\frac{P}{v} - (20000x9.81\frac{1}{100} + 128) = 0$$

$$\frac{P}{20} = 2088N$$

Power= $41760W$

3. A car of mass 1.5 metric tones moves with a constant speed of 6m/s up a slope of inclination $sin^{-1}\left(\frac{1}{7}\right)$. Given that the engine of the car is working at a constant rate of 18kW. Find the resistance to the motion

Solution



$\sin\theta = \frac{1}{7}$

Using $F = ma$

$F_T - (mgSin\theta + R_1) = ma$

$a = 0ms^{-2}$constant speed

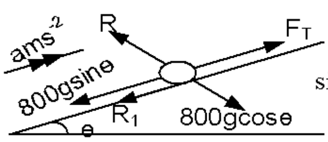
$$\frac{18000}{6} - (1500x9.81x\frac{1}{7} + R_1) = 0$$

$R_1 = 900N$

4. A car of mass 800kg with the engine working at a constant rate of 15kW climbs a hill of inclination 1 in 98 against a constant resistance to motion of 420N. Find the
   (i) Acceleration of a car up a hill when travelling with a speed of 10m/s
   (ii) Maximum speed of the car up the hill

Solution

(i)



$\sin\theta = \frac{1}{98}$

Using $F = ma$

$F_T - (mgSin\theta + R_1) = ma$

$$\frac{15000}{10} - (800x9.81x\frac{1}{98} + 420) = 800a$$

$a = 1.25ms^{-2}$

(i) $F_T - (mgSin\theta + R_1) = ma$

$a = 0ms^{-2} maximum\ speed$

$$\frac{15000}{v} - (800x9.81x\frac{1}{98} + 420) = 0$$

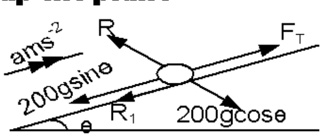
$v = 30m/s$

5. The maximum power developed by the engine of a car of mass 200kg is 44kW. When the car is travelling at 20kmh^{-1} up an incline of 1 in 8 it will accelerate at 2ms^{-2} . At what rate will it accelerate when travelling down an incline of 1 in 16 at 60kmh^{-1}. If in both cases the engine is developing the maximum power and the resistance to motor is the same.

Solution

Case I : up the plane

(i)



$\sin\theta = \frac{1}{8}$

$v = 20kmh^{-1} = \frac{20x1000}{3600} = 5.5556ms^{-1}$

Using $F = ma$

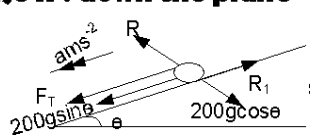
$F_T - (mgSin\theta + R_1) = ma$

$$\frac{44000}{5.5556} - (200x9.81x\frac{1}{8} + R_1) = 200a$$

$R_1 = 7274.75N$

Retarding force= $7275N$

Case II : down the plane



$\sin\theta = \frac{1}{16}$

$$v = \frac{60x1000}{3600} = 16.6667ms^{-1}$$

Using $F = ma$

$F_T + (mgSin\theta - R_1) = ma$

$$\frac{44000}{16.6667} + (200x9.81x\frac{1}{16} - 7275) = 200a$$

$a = -22.56ms^{-2}$

PUMP RAISING AND EJECTING WATER.

Consider a pump which is used to raise water from a source and then eject it at a given speed. The total work done is sum of potential energy in raising the water and kinetic energy given to the water. The work done per second gives the rate (power) at which the pump is working.

$$\boxed{work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second}$$

Example

1. A pump raises water through a height of 3.0m at a rate of 300kg per minute and delivers it with a velocity of 8.0ms-1. Calculate the power output of the pump

Solution

$$\boldsymbol{work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second}$$

$$\boldsymbol{work\ done\ per\ second} = (mass\ per\ secondxgxh) + \left(\frac{1}{2}xmass\ per\ secondxv^2\right)$$

$$\boldsymbol{work\ done\ per\ second} = \left(\frac{300}{60}x9.81x3\right) + \left(\frac{1}{2}x\frac{300}{60}x(8)^2\right) = \boldsymbol{310J}$$

2. A pump draws 3.6$m^3$of water of density 1000kgm^{-3} from a well 5m below the ground in every minute, and issues it at ground level r a pipe of cross-sectional area 40cm^2. Find
   (i) The speed with which water leaves the pipe
   (ii) The rate at which the pump is working
   (iii) If the pump is only 80% efficient, find the rate at which it must work
   (iv) Find the power wasted

Solution

ii) $volume\ per\ second = area\ x\ velocity$

$$\frac{3.6}{60} = 40x10^{-4}\ v$$

$$v = 15\ ms^{-1}$$

iii) Mass per second =volume per second $x\rho = \frac{3.6}{60}x1000 = 60kgs^{-1}$

$$work\ done\ per\ second = P.E\ given\ to\ water\ per\ second + K.E\ given\ to\ water\ per\ second$$

$$work\ done\ per\ second = (mass\ per\ secondxgxh) + \left(\frac{1}{2}xmass\ per\ secondxv^2\right)$$

$$work\ done\ per\ second = (60x9.81x5) + \left(\frac{1}{2}x60x15^2\right) = 9693W$$

iv) Efficiency $= \frac{power\ output}{power\ input}x100\%$

$$80\% = \frac{9693}{power\ input}x100\%$$

$$power\ input = 12116.25W$$

v) Power wasted= power output – power input

Power wasted= $12116.25 - 9693 = 2423.25W$

EXERCISE:13

1. A man of mass 75kg climbs 300m in 30 minutes. At what rate is he working **An[125W]**
2. A pump with a power output of 600W raises water from a lake a height of 3.0m and delivers it with a velocity of 6.0ms^{-1}. What mass of water is removed from the lake in one minute **An[7500kg]**
3. What is the power output of a cyclist moving at a steady speed of 5.0ms^{-1} along a level road against a resistance of 20N **An[100W]**
4. What is the maximum speed which a car can travel along road when its engine is developing 24kW and there is a resistance to motion of 800N **An[30ms^{-1}]**
5. A crane lifts an iron girder of mass 400kg at a steady speed of 2.0ms^{-1}. At what rate is the carne working **An[8000W]**
6. A man of mass 70kg rides a bicycle of mass 15kg at a steady speed of 4.0ms^{-1} up a road which rises 1.0m fro every 20m of its length. What power is the cyclist developing if there is a constant resistance to motion of 20N **An[250W]**

7. A lorry of mass 2000kg moving at 10m/s on a horizontal surface is brought to rest in a distance of 12.5m by the brakes being applied.
    (a) Calculate the average retarding force
    (b) What power must the engine produce if the lorry is to travel up a hill of 1 in 10 at a constant speed of 10m/s, frictional resistance being 200N. **An[8000N, 22000W]**
8. A car of mass 900kg travelling at 30m/s along a level road is brought of rest in a distance of 35m by its brakes.
    (a) Calculate the average exerted by the brakes
    (b) If the same car travels up a slope of 1 in 15 at a constant speed of 25m/s, what power does the engine develop if the total frictional resistance is 120N
9. A bullet of mass 50g travelling horizontally at $500ms^{-1}$ strikes as stationary block of wood and after travelling 10cm, it emerges from the block travelling at $100ms^{-1}$. Calculate the average resistance of the block to the motion of the bullet. **An[60000N]**
10. A horizontal force of 2000N is applied to a vehicle of mass 400kg which is initially at rest on a horizontal surface. If the total force opposing the motion is common at 800N, calculate;
    (a) The acceleration of the vehicle
    (b) The kinetic energy of the vehicle 5s after the force is first applied
    (c) The total power developed 5s after the force is first applied **An[$3.0m/^2$, 45kJ, 30kW]**
11. A lorry of mass $3.5x10^4$kg attains a steady speed v while climbing an incline of 1 in 10, with the engine operating at 175kW. Find v (neglect friction) **An[5.0m/s]**
12. A point A is vertically below the point B. A particle of mass 0.1kg is protected from A vertically up wards with a speed $21ms^{-1}$ and passes through point B with speed $7ms^{-1}$. Find the distance from A to B **An[20m]**
13. The friction resistance to the motion of a car of mass 100kg is 30VN where V is the speed in ms^1. Find the steady speed at which the car ascends a hill of inclination $\sin^{-1}(\frac{1}{10})$. If the power exerted by the engine is 12.8kW. **An[V=10m/s]**
14. A load of 3Mg is being hauled by a rope up a slope which rises 1 in 140. There is a retardation force due to friction of 20gN per Mg at a certain instant when the speed is $16kmh^{-1}$ and the acceleration is $0.6ms^{-2}$. Find the pull in the rope and the power exerted at the instant. **An[2598N, 11.55kW]**
15. A car of mass 2 tones moves from rest down. A road of inclination $\sin^{-1}(\frac{1}{20})$ to the horizontal. Given that the engine develops a power of 64.8kW when it is travelling at a speed of $54kmh^{-1}$ and the resistance to motion is 500N, find the acceleration. **An[$2.4m/s^2$]**
16. A car is driven at a uniform speed of $48kmh^{-1}$ up a smooth incline of 1 in 8. If the total mass of the car is 800kg and the resistance are neglected calculate the power at which the car is working. **An[$1.31x10^4$W]**
17. A train whose mass is 250Mg run up an incline of 1 in 200 at a uniform rate of 32km/h. The resistance due to friction is equal to the weight of 3Mg. at what power is the engine working?. **An[370.2kW]**
18. A train of mass $1x10^5$kg acquires a uniform speed of $48kmh^{-1}$from rest in 400m. Assuming that the frictional resistance of 300gN. Find the tension in the coupling between the engine and the train. And the maximum power at which the engine is working during 400m run, the mass of the engine may be neglected. **An[25162N, 335.5kW]**
19. A car of mass 2000kg travelling at $10ms^{-1}$ on a horizontal surface is brought to rest in a distance of 12.5m by the action of its brakes. Calculate the average retarding force. What power must the engine develop in order to take the vehicle up an incline of 1 in 10 at a constant speed of $10ms^{-1}$ if the frictional resistance is equal to 2000N. **An[8000N, 21600N]**
20. A water pump must work at a constant rate of 900W and draws $0.3m^3$ of water from a deep well and issue it through a nozzle situated 10m above the level from which the water was drawn after every minute. If thee pump is 75% efficient, find;
    i) Velocity with which the water is ejected
    ii) The cross-sectional area of the nozzle **An (8.6ms^{-1}, $5.81cm^2$)**

UNEB 2017 No1c

A bullet of mass 10g moving horizontally with a velocity of 300m/s into a block of wood of mass 290g which rests on a rough horizontal floor. After impact, the block and bullet move together and come to rest when the block has trvaelled a distance of 15m. calculate the coefficient of slidding friction between the block and the floor. **An**(0.34) (07marks)

UNEB 2015 No1

(a)(i) What is meant by **a conservative force** (01mark)

(ii) Give **two** examples of a conservative force (01mark)

(b)(i) State the law of conservation of **mechanical energy** (01mark)

(ii) A body of mass m, is projected vertically upwards with speed, u. Show that the law of conservation of mechanical energy is obeyed through its motion (05marks)

(iii) Sketch a graph showing variation of kinetic energy of the body with time (01mark)

(c) (i) Describe an experiment to measure the coefficient of static friction (04marks)

(ii) State two disadvantages of friction (01marks)

(d) A bullet of mass 20g moving horizontally strikes and gets embedded in a wooden block of mass 500g resting a horizontal table. The block slides through a distance of 2.3m before coming to rest. If the coefficient of kinetic friction between the block and the table is 0.3, calculate the

(i) Friction force between the block and the table (02marks)

(ii) Velocity of the bullet just before it strikes the block (04marks)

An($1.53N$, $95.68m/s$)

UNEB 2014 No3

(a) Define **work and energy** (02marks)

(b) Explain whether a person carrying a bucket of water does any work on the bucket while walking on a level road (03marks)

(c) A pump discharges water through a nozzle of diameter 4.5 cm with a speed of $62ms^{-1}$ into a tank 16 m above the intake.

(i) Calculate the work done per second by the pump in raising the water if the pump is ideal

(ii) Find the power wasted if the efficiency of the pump is 73% (02marks)

(iii) Account for the power lost in (c) (ii) (02marks)

An($2.05x10^5 J s^{-1}$, $7.6x10^4 W$)

(d) (i) State the **work-energy theorem** (01mark)

(ii) Prove the work-energy theorem for a body moving with constant acceleration.

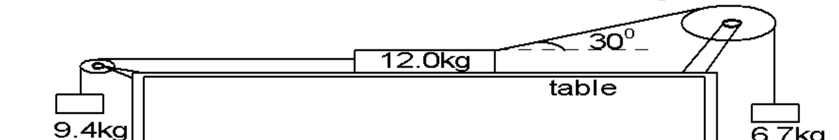
(e) Explain briefly what is meant by internal energy of a substance (03marks)

UNEB 2013 No1

(a) Using the molecular theory, explain the laws of friction between solid surface (06marks)

(b) With the aid of a labeled diagram, describe how the coefficient of static friction for an interface between a rectangular block of wood and a plane surface can be determined. (06marks)

(c) The diagram below shows three masses connected by inextensible strings which pass over smooth pulleys. The coefficient of friction between the table and the 12.0 kg mass is 0.25

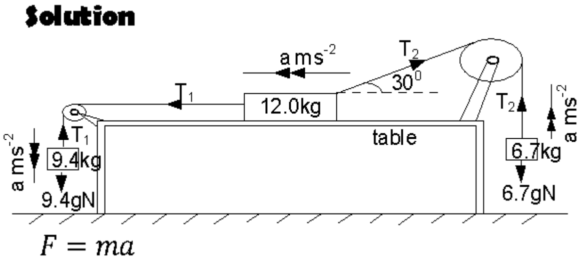


If the system is released from rest, determine the

(i) Acceleration of the 12.0kg mass (05marks)

(ii) Tension in each string (03marks)

Solution



$F = ma$

9.4kg mass: $9.4gN - T_1 = 9.4a$

$T_1 = 9.4g - 9.4a$..............(1)

For 6.7kg mass: $T_2 - 6.7gN = 6.7a$

$T_2 = 6.7a + 6.7g$...................(2)

For 12kg mass:

$T_1 - (T_2 Cos30° + 0.25R) = 12a$...........(3)

But $R + T_2 Sin30° = 12gN$

$\therefore R = 12g - T_2\,Sin30°$
put into(3)
$T_1 - (T_2\,Cos30° + 0.25[12g - T_2\,Sin30])$= $12a$
Put equation(1)
9.4g-9.4a-T_2 $Cos30°$-0.25x12g+$0.25xT_2\,Sin30 = 12a$
$a = 0.53\,m\,s^{-2}$
Acceleration of 12kg mass is $0.53\,m\,s^{-2}$

(i) Tension in each string
$T_1 = 9.4g - 9.4a$
$T_1 = 9.4x9.81 - 9.4x0.53 = 87.2N$
Also $T_2 = 6.7a + 6.7g$
$T_2 = 6.7x0.573 + 6.7x9.81 = 69.3N$

UNEB 2013 No4d

A simple pendulum of length 1m has a bob of mass 100g. it is displaced from its mean position A and to a position B so that the string makes an angle of 45° with the e vertical. Calculate the;

(i) Maximum potential energy of the bob **An(0.287J)** [03marks]

(ii) Velocity of the bob when the string makes an angle of 300 with the vertical (neglect air resistance) **An(1.766m/s)** [04marks]

UNEB2010No3

(c) i) State the laws of solid friction [03marks]

ii) With the aid of a well labeled diagram describe an experiment to determine the co-efficient of kinetic friction between the two surfaces. [05marks]

d) A body slides down a rough plane inclined at 30° to the horizontal. If the co-efficient of kinetic friction between the body and the plane is 0.4. Find the velocity after it has travelled 6m along the plane.

An[4.25m/s] [05marks]

UNEB2008 No2

a) i) state the laws of friction between solid surfaces [03marks]

ii) Explain the origin of friction force between two solid surfaces it contact. [03marks]

(iii) Describe an experiment to measure the co-efficient of kinetic friction between two solid surfaces.

b) i) A car of mass 1000kg moves a long a straight surface with a speed of 20ms^{-1}. When brakes are applied steadily, the car comes to rest after travelling 50m. Calculate the co-efficient of friction between the surface and the types. **An**[$\mu = 0.408$] [04marks]

c) ii) State the energy changes which occur from the time the brakes are applied to the time the car comes to rest. **An[kinetic energy**$\rightarrow$ $heat + sound\ energy$] [02marks]

d) i)State two disadvantages of friction [01marks]

e) ii) Give one method of reducing friction between solid surfaces. [01mark]

UNEB2007No3

a) i)State the laws of solid friction [03marks]

ii) Using the molecular theory, explain the laws stated in a i). [03marks]

b) Describe an experiment to determine the co-efficient of static friction for an interface between a rectangular block of wood and plane surface. [04marks]

c) i)State the different between conservative and non conservative forces, giving one example of each.

ii) State the work-energy theory. [01marks]

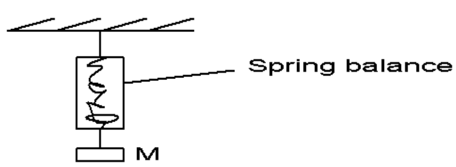
iii) A block of mass 6.0 kg is protected with a velocity of 12ms^{-1} up a rough plane inclined at 45° to the horizontal if it travels 5.0m up the plane. Find the frictional force. **An[44.8N]** [04marks]

UNEB2006No2

a) i)Define force and power [02marks]

ii)Explain why more energy is required to push a wheelbarrow uphill than on a level ground.

b)



A mass M is suspended from a spring balance as shown above. Explain what happens to the reading on the spring balance when the set up is raised slowly to a very high height above the ground. [02marks]

c) i)State the work-energy theorem [01mark]

Solution

b)As the setup is raised to a high height, acceleration due to gravity reduces, the weight of M decreases and it reading of the spring balance reduces proportionately.

UNEB2005No1

a) i)What is meant by conservation of energy? [01mark]

ii)Explain how conservation of energy applies to an object falling from rest in a vacuum. [02marks]

UNEB2004 N01

a) State the laws of friction [04marks]

b) A block of mass 5.0kg resting on the floor is given horizontal velocity of $5ms^{-1}$ and comes to rest in a distance of 7.0m. Find the co-efficient of kinetic friction between the block and the floor.

An[0.182] [04marks]

c) i)State the laws of conservation of linear momentum [01mark]

ii)What is perfectly inelastic collision? [01mark]

d) A car of mass 1500kg rolls from rest down a round inclined to the horizontal at an angle of 35°, through 50m. The car collides with another car of identical mass at the bottom of the incline. If the two vehicles interlock on collision and the co-efficient of kinetic friction is 0.20, final the common velocity of the vehicle.

An[20.05m/s] [08marks]

[Hint loss of p.e at the top=gain in k.e at the bottom + work done against friction]

e) Discuss briefly the energy transformation which occur in(d) above. [01mark]

An[Potential energy$\rightarrow$ $kinetic\ energy + sound + heat$**]**

UNEB 2001 No1

a) i)State the principle of conservation of mechanical energy. [01mark]

ii)Show that a stone thrown vertically upwards obey, the principle in (c) throughout its upward motion. [04marks]

CHAPTER 6: STATICS

Is a subject which deals with equilibrium of forces $e.g$ the forces which act on a bridge.

Coplanar forces

Those are forces acting on the same point (plane).

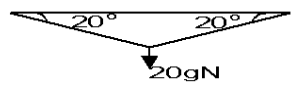
6.1.0: Conditions necessary for mechanical equilibrium

When forces act on a body then it will be in equilibrium when;

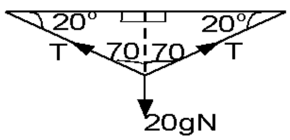
1. The algebraic sum of all forces on a body in any direction is zero
2. The algebraic sum of moments of all forces about any point is zero

Examples

1. A mass of 20kg is hang from the midpoint P of a wire as shown below. Calculate the tension in the wire take $g=9.8ms^{-1}$

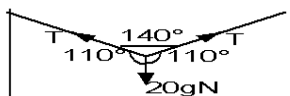


Solution



Method I : lami's theorem

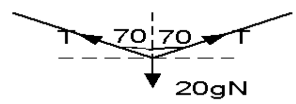
(Apply to only three forces in equilibrium)



$$\frac{20gN}{Sin140} = \frac{T}{Sin110}$$

$$T = \frac{20x9.81sin110}{Sin140} = 286.83N$$

METHOD II: Resolving



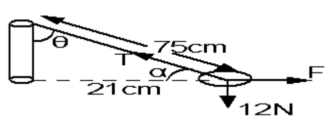
Resolving vertically

$$Tsin70 + Tcos70 = 20gN$$

$$T = \frac{20x9.81}{2\ cos70} = 286.83N$$

2. One end of a light in extensible string of length 75cm is fixed to a point on a vertical pole. A particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force. Find the magnitude of the force and the tenion in the string

Solution

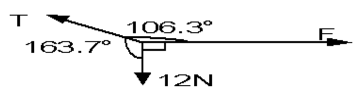


$Sin\theta = \frac{21}{75}$ $\therefore \theta = 16.3°$

Also $cos\ \alpha = \frac{21}{75}$

$\alpha = 73.7°$

Using Lami's theorem



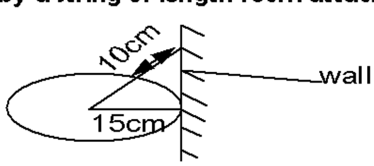
$$\frac{F}{Sin163.7} = \frac{12}{Sin106.3}$$

$$F = 3.51N$$

Also $\frac{T}{Sin90} = \frac{12}{Sin106.3}$

$T = 12.5N$

3. A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.

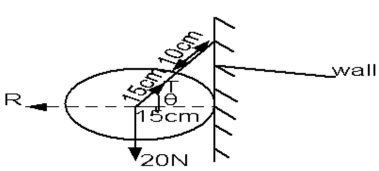


i) copy the diagram and show the forces acting on the sphere

ii) Calculate the reaction on the sphere due to the wall.

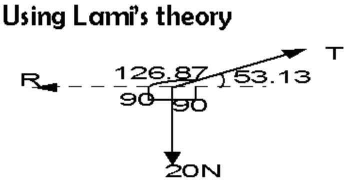
iii) Find the tension in the string

Solution



$Cos\theta = \frac{15}{28}$ $\therefore \theta = 53.13°$

Using Lami's theory



$$\frac{20}{Sin126.87} = \frac{T}{Sin90}$$

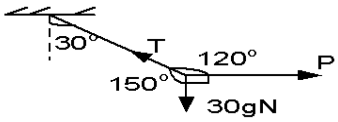
$$T = 25N$$

$$\frac{R}{Sin143.13} = \frac{20}{Sin126.87}$$

$$R = 15N$$

4. A mass of 30kg hangs vertically at the end of a light string. If the mass is pulled aside by a horizontal force P so that the string makes an angle 30° with the vertical. Find the magnitude of the force P and the tension in the string.

Solution

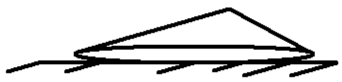


$$\frac{30x9.81}{Sin120} = \frac{20}{Sin150}$$
$P = 169.91N$

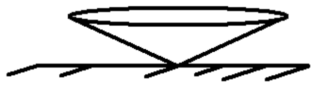
$$\frac{T}{Sin90} = \frac{30x9.81}{Sin120}$$
$T = 339.83N$

6.1.1: Types of equilibrium

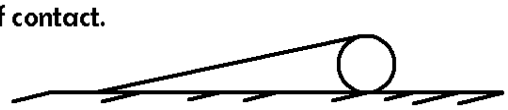
1. **Stable equilibrium.**
Stable equilibrium is when a body returns to its original position after being displaced slightly and its center of gravity rises. A body under stable equilibrium has Large base area, the center of gravity is in the lowest position.



2. **Unstable equilibrium.**
Un Stable equilibrium is when a body does not return to its original position after being displaced slightly and its center of gravity is lowered. A body under un stable equilibrium has Low base area, the center of gravity is in the highest position.



3. **Neutral equilibrium.**
The body is said to be in a neutral equilibrium if the center of gravity is neither raised nor lowered during displacement and the body remains in the displaced position.
A body under neutral equilibrium has a small area of contact The center of gravity is always at the same height directly above the point of contact.



6.3.4: CENTER OF GRAVITY

This point where the resultant force on the body due to gravity acts.

DETERMINATION OF CENTRE OF GRAVITY OF AN IRREGULAR LAMINA

- Make three holes near the edge of the card board
- Suspend the sheet form one hole and allow it to swing freely
- Hung a pendulum bob form the same point of suspension
- Trace the outline of the pendulum on the sheet
- Repeat the procedure above using the other holes.
- The point of intersection of the three outlines is the centre of gravity of the board

Definition: A uniform body is one whose center of gravity is the same point as its geometrical centre

6.2.1: Moment of a force

This is the product of a force and the perpendicular distance of its line of action from the pivot.
The unit of a moment is Nm and it's a vector quantity.
Moment of a force = Force x perpendicular distance of its line of action from pivot.

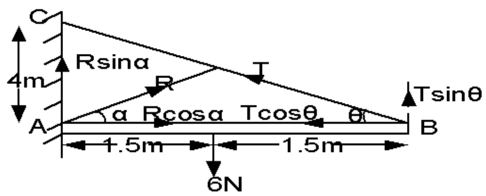
6.2.2: Principle of moments

It states that when a body is in mechanical equilibrium, the sum of clockwise moments about a point is equal to the sum of anticlockwise moments about the same point.

6.2.3: Beams hinged against the wall

1. A Uniform beam AB, 3.0m long and of weight 6N is hinged at a wall at A and is held stationary in a horizontal position by a rope attached to B and joined to a point C on the wall, 4.0m vertically above A. Find
   (i) the tension T in the rope
   (ii) the magnitude and direction of the Reaction R at the hinge.

Solution



tan$\theta = \frac{4}{3}$ $\quad\theta$= 53.13°

Taking moments about A at equilibrium

$(Tsin53.13)\, x\, 3\ =\ 9$

$T\ =\ 3.75N$

$(\uparrow)\ Rsin\alpha\ +\ Tsin\theta\ =\ 6$

$Rsin\alpha =\ 6 - 3.75sin53.13$

$Rsin\alpha\ =\ 3$------------------------------i

$(\rightarrow)\ Rcos\alpha\ =\ Tcos\theta$

$Rcos\alpha\ =\ 3.75cos53.13$

$Rcos\alpha = 2.238$----------------------------ii

i/ii tan$\alpha = \frac{3}{2.238}$ $\quad \alpha\ =\ 53.3°$

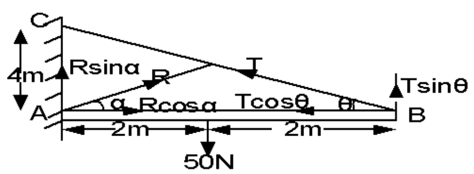
Put into 1; $Rsin53.3\ = 3$

$R\ =\ 3.74N$

The reaction at A is 3.74 at 53.28° to the beam

2. A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical wall and is held horizontal in equilibrium by a string which has one end attached at B and the other end attached to a point C on the wall, 4m above A. find
   (i) the tension T in the rope
   (ii) the magnitude and direction of the Reaction R at the hinge.

Solution



tan$\theta = \frac{4}{4}$ $\quad \therefore \theta\ =\ 45°$

Taking moments about A

$Tsin45x4\ =\ 50x2$

$T\ =\ 35.36N$

$(\uparrow)$: $Rsin\ \alpha\ +\ Tsin\ \theta\ =\ 50$

$Rsin\ \alpha + \ 35.36sin45\ =\ 50$

$Rsin\ \alpha\ =\ 24.997$..........................(i)

$(\rightarrow)$:$Rcos\ \alpha\ =\ Tcos\ \theta$

$Rcos\ \alpha\ =\ 35.36cos45$

$Rcos\ \alpha\ =\ 25$..........................(ii)

(i)/(ii) $tan\ \alpha = \frac{24.997}{25} \therefore\ \alpha\ =\ 45°$

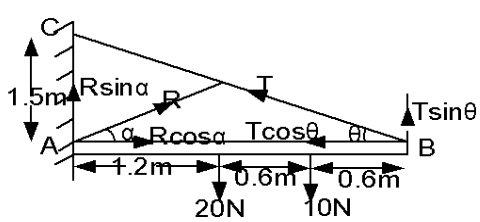
Put into (ii); $Rcos\ \alpha\ =\ 25$

$Rcos45\ =\ 25$

$R\ =\ 35.36N$ at $45°$ to the beam

3. A uniform beam AB of mass 20kg and length 2.4m is hinged at a point A in a vertical wall and is maintained in a horizontal position by means of a chain attached to B and to point C in a wall 1.5m above. If the bar carries a load of 10kg at a point 1.8m from A. calculate.
   i) The tension in the chain
   ii) The magnitude and direction of the reaction between the bar and the wall

Solution



tan$\theta = \frac{1.5}{2.4}$ $\quad \therefore\ \theta$ = 32.01°

Taking moments about A

$Tsin\ \theta\ x\ 2.4\ =\ 20gNx1.2\ +\ 10gN\ x\ 1.8$

$Tx2.4sin32.01 = 20x9.8x1.2\ +\ 10x9.8x1.8$

$T\ =\ 323.87N$

Tension in the chain $=\ 323.87N$

(ii) Reaction at the wall

$(\uparrow)\ R\ sin\ \alpha\ +\ Tsin\theta\ =\ 20gN\ +\ 10gN$

$R\ sin\ \alpha\ +\ 323.87sin32.01\ =\ 30gN$

$R\ sin\ \alpha\ =\ 122.63$.......................(i)

$(\rightarrow)$; $R\ cos\ \alpha$ = Tcosθ

$R\ cos\ \alpha\ =\ 32.87cos32.01$

$R\ cos\ \alpha\ =\ 274.63$ ....(ii) (i)/(ii) $tan\,\alpha = 0.446528055$

$\alpha = 24.1°$ Put α in eqn (ii)

$R\,cos24.1 = 274.63$

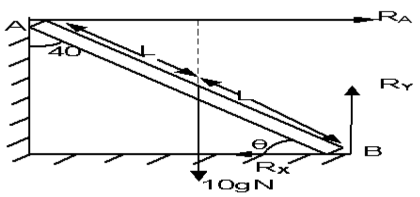
$R = 300.85N$

Reaction at A is 300.85 at 24.1° to the horizontal

6.2.4: Ladder problems

1. A uniform rod AB of mass 10kg is smoothly hinged at B and rests in a vertical plane with the end A against a smooth vertical wall. If the rod makes an angle of 40° with the wall, find the reaction on the wall and the magnitude of the reaction at B

Solution



let length of the ladder be 2L

$\theta = 90° - 40° = 50°$

Taking moments about B

$R_A\, x\, 2Lsin50 = 10x9.81lcos50$

$R_A = 41.16N$

(↑): $R_Y = 10gN = 10x9.81 = 98.1N$

(→): $R_X = R_A$

: $R_X = 41.16$

$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{(41.16)^2 + (98.1)^2}$

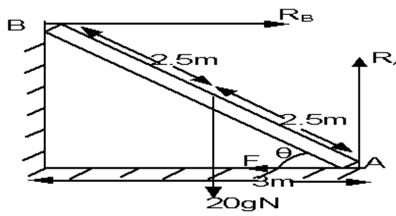
$R = 106.38N$

$\alpha = tan^{-1}\left(\frac{R_Y}{R_X}\right) = tan^{-1}\left(\frac{98.1}{41.16}\right) = 67.24°$

Reaction at B is $106.38N$ at $67.24°$ to the beam.

2. uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate the functional force between the ladder and the ground and the coefficient of friction

Solution



$Cos\theta = \frac{3}{5}$ ∴ θ = 53.13°

Resolving vertically: $R_A = 20gN$

$R_A = 20x9.81 = 196.2N$

Taking moments about A

$R_B\, x\, 5sin\theta = 20x9.81x2.5cos\theta$

$R_B\, 5sin53.13 = 20x9.81x2.5cos53.13$

$R_B = 73.56N$

Resolving horizontally: R_B= F

$F = 73.56N$

But $F = \mu x R_A$

$73.56 = \mu x196.2$

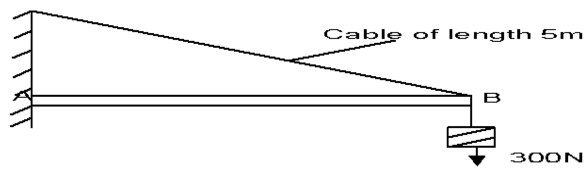
$\mu = 0.37$

Exercise: 14

1. A particle whose weight is 50N is Suspended by a light string which is 35°° to the vertical under the action of horizontal force F. Find
   (a) The tension in the string
   (b) Force F **An(61.0N, 35.0N)**
2. A particle of weight W rests on a smooth plane which is inclined at 40° to horizontal. The particle is prevented from slipping by a force of 50.0N acting parallel to the plane and up a line of greatest slope. Calculate
   (a) W
   (b) Reaction due to the plane **An(77.8N, 59.6N)**
3. Two light strings are perpendicular to each other and support a particle of weight 100N. the tension in one of the strings is 40.0N. Calculate the angle this string makes with the vertical and the tension in the other string **An(66.4°, 91.7N)**
4. A uniform pole AB of weight 5W and length 8a is suspended horizontally by two vertical strings attached to it at C and D where AC=DB=a. A body of weight 9W hangs from the pole at E where ED=2a. calculate the tension in each string **An(5.5W, 8.5W)**
5. AB is a uniform rod of length 1.4m. It is pivoted at C, where AC=0.5m, and rests in horizontal equilibrium when weighs of 16N and 8N are applied at A and B respectively. Calculate
   (a) the weight of the rod

(c) (b) the magnitude of the reaction at the pivot **An(4N, 28N)**

6. A uniform rod AB of length 4a and weight W is smoothly hinged at its upper end, A. the rod is held at 30° to the horizontal by a string which is at 90° to the rod and attached to it at C where AC=3a, find
   (d) the tension in the string
   (e) reaction at A **An(0.58W, 0.578W)**
7. A sphere of weight 40N and radius 30cm rests against a smooth vertical wall. The sphere is supported in this position by a string of length 20cm attached to a point on the sphere and to the a point on the wall. Find
   (a) tension in the string
   (b) reaction due to the wall **An(50N, 30N at 90° to the wall)**
8. A uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and i6ts lower end on rough ground. The bottom of the ladder is 3m from the wall. Calculate the frictional forces between the ladder and ground **An(75N)**
9. One end of a uniform plank of length 4m and weight 100N is hinged to the vertical wall. An inelastic rope, tied to the other end of the plank is fixed at a point 4m above the hinge. Find
   i. The tension in the rope'
   ii. The reaction of the wall on the plank **An(388.9N, 302.1N at 24.4° to horizontal)**
10.



The figure shows a uniform rod AB of weight 200N and length 4m, the beam is hinged to the wall at A.
   i. Find the tension in the cable
   ii. The horizontal and vertical components of the force exerted on the beam by the wall
   iii. The reaction of the wall on the beam at point A

   An(666.7N, 533.3N, 99.98N, 542.59 at 10.6° to the horizontal)
11. A uniform beam AB of length 2L rests with end A in contact with a rough horizontal ground. A point C on the beam rests against a smooth support. AC is of length $\frac{3l}{2}$ with C higher than A and AC making an angle of 60° with the horizontal. If the beam is in limiting equilibrium, find the coefficient of friction between the beam and the ground.
12. A uniform ladder of mass 25kg rests in equilibrium with its base on a rough horizontal floor and its top against a smooth vertical wall. If the ladder makes an angle of 75° with the horizontal, find the magnitude of the normal reaction and of the frictional force at the floor and state the minimum possible value of the coefficient of friction μ between the ladder and the floor.
13. A ladder 12m long and weighing 200N is placed 60° to the horizontal with one end B leaning against the smooth wall and the other end A on the ground. Find;
   a) reaction at the wall **An(57.7N)**
   b) reaction at the ground **An(208.2N at 73.9° to the horizontal).**

6.3.0: Couples

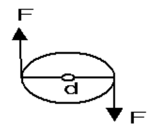
A couple is a pair of **equal**, **parallel** and **opposite** forces with different lines of action acting on a body.

Examples

- Forces in the driver's hands applied to a steering wheel
- Forces in the handles of a bike
- Forces in the peddles of a bike
- Forces experienced by two sides of a suspended rectangular coil carrying current in a magnetic field.

6.3.1: Moment of a couple (torque of a couple)

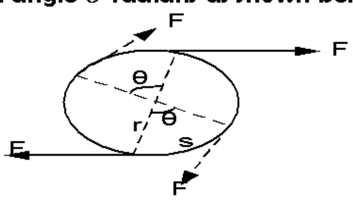
It is defined as the product of one of the forces and the far distance between the lines of action of the forces



Moment of a couple or torque of couple = F x d

6.3.2: Work done by a couple

Consider two opposite and equal forces acting tangentially on a wheel of radius r, suppose the wheel rotates through an angle θ radians as shown below.



Work done by each force = FxS

But $s = \frac{\theta}{360} x2\pi r$

360° = 2π $rads$

Work done by each force = $F\ x\ r\theta$

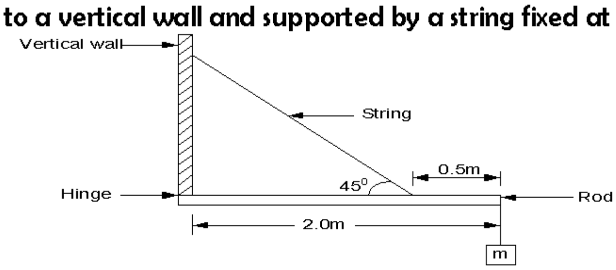
Total work done by the couple = $2Fr\theta$

UNEB 2015 No 2

(a) (i) State the **principle of moments** (1marks)

(ii) Define the terms **center of gravity** and **uniform body** (2marks)

(b) The figure below shows a body, m of mass 20kg supported by a rod of negligible mas horizontally hinged to a vertical wall and supported by a string fixed at 0.5m from the other end of the rod



Calculate the

(i) Tension in the string (3marks)

(ii) Reaction at the hinge (3marks)

(iii) Maximum additional mass which can be added to the mas of 20 kg before the string can break given that the string cannot support a tension of more than 500N (2marks)

An(370N,270N,7.03kg)

UNEB 2009 No 2

a) Define the following terms

i) Velocity (2marks)

ii) Moment of a force

c)(i) State the condition necessary for mechanical equilibrium t be attained.(2 marks)

ii)A uniform ladder of mass 40kg and length 5m, rest with its upper end against a smooth vertical wall and with its lower end at 3m from the wall on a rough ground. Find the magnitude and direction of the force exerted at the bottom of the ladder **An**[418.7N at an angle of 69.4° to the horizontal]**.** (06 marks)

UNEB 2006 No 2

c) State the condition for equilibrium of a rigid body under the action of coplanar forces. (2mk)

d) A 3m long ladder at an angle 60° to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5kg and its centre of gravity is one third from the bottom of the ladder.

i) Draw a sketch diagram to show the forces acting on the ladder. (2mk)

ii) Find the reaction of the ground on the ladder. (4mk)

(Hint Reaction on the ladder = $\sqrt{R^2 + F^2}$ **) An(49.95N at 79.11° to the horizontal)**

UNEB 2006 No1

e) Describe an experiment to determine the centre of gravity of a plane sheet of material having an irregular shape. (4 marks)

UNEB 2005 No2

f) (i) Define centre of gravity (1 mark)

(ii) Describe an experiment to find the centre of gravity of a flat irregular card board. (3 marks)

UNEB 2002 No2

d) (i)Define moment of a force (1 mark)

(ii) A wheel of radius 0.6m is pivoted at its centre. A tangential force of 4.0N acts on the wheel so that the wheel rotates with uniform velocity find the work done by the force to turn the wheel through 10 resolutions.

Solution

Work done $= force\ x\ distances$

But distance= circumference x number of revolutions

$= 2\pi\, rx10$

$W = F\ x\ d = 4x2\,\pi\, rx0.6x10$

$W\ = 150.79J$

UNEB 2000 No3

b) State the conditions for equilibrium of a rigid body under the action of coplanar forces.(2mk)

d) A mass of 5.0kg is suspended from the end A of a uniform beam of mass 1.0kg and length 1.0m. The end B of the beam is hinged in a wall. The beam is kept horizontal by a rope attached to A and to a point C in the wall at a height 0.75m above B

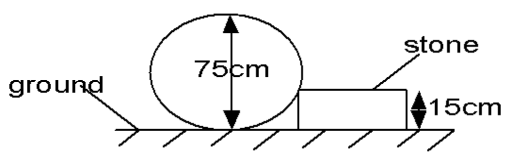
i. Draw a diagram to show the forces on the beam (2 marks)
ii. Calculate the tension in the rope (4 marks)
iii. What is the reaction exerted by the hinge on the beam (5 marks)

An (89.8N, 72.01N, at 3.95° to the beam)

UNEB 1998 No1

d) (i)Explain the term unstable equilibrium (3mk)

(ii) An oil drum of diameter 75cm and mass 90kg rests against a stone as shown



Find the least horizontal force applied through the centre of the drum, which will cause the drum to roll up the stone of height 15cm.

An(1177.2N) (5 marks)

CHAPTER 7: CIRCULAR MOTION

This is the motion of the body with a uniform speed around a circular path of fixed radius about a center.

Terms used in circular motion

Consider a body of mass m initially at point A moving with a constant speed in a circle of radius r to point B in a time Δt, the radius sweeps out an angle $\Delta\theta$ at the centre

1. **Angular velocity (ω)**

This is the is the rate of change of the angle for a body moving in a circular path.

Or rate of change of angular displacement i.e $\omega = \frac{\Delta\theta}{\Delta t}$

For large angles and big time intervals. $\omega = \frac{\theta}{t}$

Angular velocity is measured in radians per second ($rads^{-1}$)

2. **Linear speed (v)**

If the distance of the arc AB is, s and the speed is constant then velocity.

$v = \frac{Arc\ length}{time} = \frac{s}{\Delta t}$

But $s = \frac{\Delta\theta}{360}\ x 2\pi r = \Delta\theta r$

Since $360° = 2\pi$ rads

$\therefore\ v = \frac{\Delta\theta r}{\Delta t} = r\,\omega$

Where $\frac{\Delta\theta}{\Delta t} = \omega$

$\boxed{v = r\,\omega}$ –units are ms^{-1}

Definition

Velocity is the rate of change of displacement for a body moving a round a circular path about a fixed point or centre.

3. **Period T**

This is the time taken for the body to describe one complete are revolution

$T = \frac{Circumference[distance\ around\ a circle]}{velocity} = \frac{2\pi r}{v}$

But $v = r\,\omega$

$T = \frac{2\pi r}{\omega r}$

$\boxed{T = \frac{2\pi}{\omega}}$ units seconds.

1. **Acceleration**

Centripetal acceleration is defined as the rate of change of velocity of a body moving in a circular path and is always directed towards the centre.

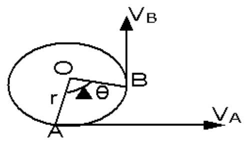
7.1.0: Derivation of $a = \frac{v^2}{r}$

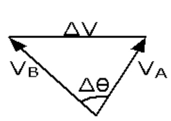
Question:

Show that the acceleration of a body moving round a circular path with speed v is given by $\frac{v^2}{r}$ where r is the radius of the path.

Solution

Consider a body of mass m moving around a circular path of radius r with uniform angular velocity ω and speed V. If initially the body is at point A moving with velocity V_A and after a small time internal Δt, the body is at point B where its velocity is V_B with the radius having moved an angle $\Delta\theta$





Acceleration, $a = \frac{change\ in\ velocity}{time} = \frac{V_B - V_A}{\Delta t}$

but $V_B - V_A = V\Delta\theta$

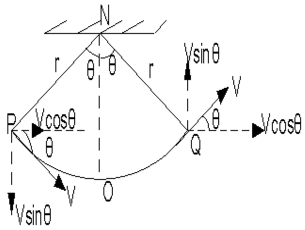
$a = \frac{V\Delta\theta}{\Delta t}$

$\frac{\Delta\theta}{\Delta t} = \omega = \frac{v}{r}$

$\boxed{a = \frac{v^2}{r}}$

Question

A volume of mass m is oscillated from a fixed point by a string of length r with a constant speed V. Shows that the acceleration of the body is $\frac{V^2}{r}$ and directed towards the centre.



Acceleration a= $\frac{change\ in\ velocity}{time}$

Horizontal component

$$a_X = \frac{vCos\theta - vCos\theta}{t}$$

$a_x = \frac{0}{t}$

a_x =0 ,the body does not accelerate horizontally.

Vertical component

$a_y = \frac{vSin\theta - -vSin\theta}{t}$

$a_y = \frac{2vSin\theta}{t}$ ------------(1)

but time (t) $= \frac{Arc\ length\ PQ}{Velocity}$

$t = \frac{2\theta}{360} x \frac{2\pi r}{v} = \frac{2\theta x 2\pi r}{2\pi v}$

where $360° = 2\pi rads$

$t = \frac{2\theta r}{V}$

put into equation (1)

$a_y = \frac{2v sin\theta}{\left(\frac{2\theta r}{v}\right)} = \frac{v^2 Sin\theta}{r\theta}$

for small angle sinθ $\approx \theta$

$a_y = \frac{v^2\theta}{r\theta}$

$$\boxed{a_y = \frac{v^2}{r}}$$

EXAMPLE

1. A particle moves along a circular path of radius 3.0m with an angular velocity of 20 rad s^{-1} calculate;
   a) The linear speed of the particle
   b) Angular velocity in revolutions per second
   c) Time for one revolution
   d) The centripetal acceleration

Solution

$r = 3m$ $\omega = 20$ rads^{-1}

a) Linear speed $v = r\omega$

$v = 20x3$ =60ms^{-1}

b) Angular velocity in rev per second gives the frequency

$\omega = 2\pi f$ $\therefore$ $f = \frac{\omega}{2\pi}$

$f = \frac{20}{2\pi} = 3.18$ rev per second

c) Time for one revolution (T)

$T = \frac{1}{f} = \frac{1}{3.18} = 0.31\ s$

d) Acceleration $a = \frac{v^2}{r}$

$a = \frac{60^2}{3} = 1200ms^{-2}$

2. A body is fixed on the string and whirled in a circle of radius 10cm. If the period is 5s. find
   i) The angular velocity
   ii) The speed of the body in the circle
   iii) The acceleration of the body
   iv) The frequency

Solution

i) $\omega = \frac{\theta}{t}$

its whirled in a circle

$(\theta = 360° = 2\pi)$

$\omega = \frac{2\pi}{t} = \frac{2x\frac{22}{7}}{5} = 1.26$rads^{-1}

ii) $v = \omega$ r

$v = 1.26x\frac{10}{100} = 0.13$ms^{-1}

iii) $a = \omega^2 r$

$a = (1.26)^2 x0.1 = 0.169ms^{-2}$

iv) $f = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{1.26} = 0.2Hz$

EXERCISE:15

1. A particle of mass 0.2kg moves in a circular path with an angular velocity of 5 rads^{-1} under the action of a centripetal force of 4N. What is the radius of the particle. **An(0.8m).**
2. Calculate the tension in the wire of hammer throwers when a hammer of mass 7kg is being swung round at 1 rev per second in a circle of radius 1.5m **An(414N)**
3. What force is required to cause a body of mass 3g to move in a circle of radius 2m at a constant rate of 4 revolutions per second. **An(3.8N)**
4. A particle moves along a circular path of radius 3m with an angular velocity of 20. Calculate the
   (i) Linear speed of particle
   (ii) Angular velocity in revolutions per second
   (iii) Time taken for one revolution. **An(60m**$s^{-1}, 3.2rev\ s^{-1}, 0.31s$**)**
5. An astronaut is trained in a centrifuge that has an arm of length 6m. if the astronaut can stand an acceleration of 9g ms^{-2}, what is the maximum number revolutions per second that the centrifuge may make?

7.1.1: CENTRIPETAL AND CENTRIFUGAL FORCES

If a body is moving in a circle, it will experience an initial outward force called **centrifugal force**. These forces always act away from the center and are perpendicular to the direction of motion.
In order for the body to continue moving in a circle without falling off, there must be an equal and opposite force to the centrifugal force. This force which counter balances the centrifugal force is called the **centripetal force** and always acts towards the center of the motion.

Definition

Centripetal force is an inward force towards the center of the circle required to keep a body moving in a circular path
If the mass of the body is m then the centripetal force
F =ma

But a = $\frac{v^2}{r}$

$\boxed{F = \frac{m\,v^2}{r}}$ This is the expression for the centripetal force Or $\boxed{F = mr\,\omega^2}$

Question

Explain why there must be a force acting on a particle which is moving with uniform speed in a circular path. Write down an expression for its magnitude.

Solution

- If a body is moving along circular path, there must be a force acting on it, for if there were not, it would move in a straight line in accordance with Newton's first law.
- Since the body is moving with a constant speed, this force cannot at any stage have a component in direction of motion of a body. For it did, it would increase or decrease the speed of the body. The force on the body must therefore be perpendicular to direction of motion and directed towards the center.

7.1.2: Examples of centripetal forces

1. **A car moving around a circular truck**: For a car negotiating a corner or moving on a circular path, the frictional force between the wheels and the surface provides the necessary centripetal force required to keep it on the track.

2. **A car moving on banked track**:
   For a banked track, the centripetal force is provided by the frictional force and the horizontal components of the normal reaction.

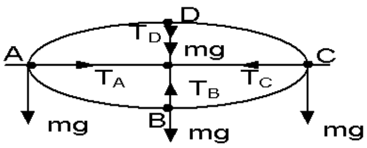
3. a) **Tension on the string keeping a whirling body in a vertical circle**.
   The tension force in the string provides the necessary centripetal force
   b) **For the conical pendulum, the horizontal component of the tension in the string** provides the necessary centripetal force

4. **Gravitational force on planets**
   For a planet orbiting round the sun or satellite revolving about the earth, the gravitational force between the two bodies provides the necessary centripetal force required to keep the satellite in the orbit.

5. **Electrostatics force on the electrons**
   For electrons moving round the nucleus, the electrostatics force provides the necessary centripetal force.

7.1.3: Motion in a vertical cycle

Consider a body of mass m attached to a string of length r and whirled in a vertical circle with a constant speed V. If there is no air resistance to the motion, then the net force towards the centre is the centripetal force.



At point A: $T_A = \frac{m\,v^2}{r}$ ------------------------(2)

At point B: $T_B = \frac{m\,v^2}{r} + mg$ - ----------------(3)

At point C: $T_C = \frac{m\,v^2}{r}$--------------------------(4)

At point D: $T_D = \frac{m\,v^2}{r} - mg$ ------------------(5)

The maximum tension in the vertical circle is experienced at B

$T_{max} = \frac{m\,v^2}{r} + mg$

The minimum tension is experienced on the top of the circle at point D

$T_{min} = \frac{m\,v^2}{r} - mg$

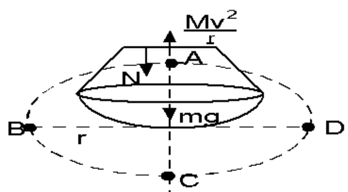
Note

If the speed of whirling is increased the string will most likely break at the bottom of the circle. Motion is tangential to the circle and when string breaks the mass will fly in a parabolic path.

Question

Explain why a bucket full of water can be swung round a vertical circle without spilling.

Solution

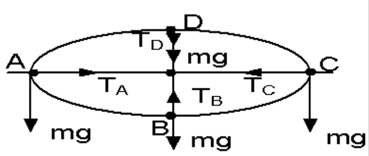


When the bucket is inverted vertically above the point of support, the weight of the water is less than the required centripetal force, the reaction at bucket base on the water proiveds the rest of the centripetal force so the water stays in the bucket

Examples

1. An object of mass 3kg is whirled in a vertical circle of radius 2m with a constant sped of 12ms^{-1}, calculate the maximum and minimum tension in the string

Solution



Maximum tension is at B

$T - mg = \frac{m\,v^2}{r}$

T = $\frac{3x12^2}{2}$ $+ 3x9.81 = 245.43N$

Minimum tension is at D

T= $\frac{m\,v^2}{r} - mg$

$T = \frac{3x12^2}{2} - 3x9.81$

T = $186.57N$

2. A stone of mass 800g is attached to string of length 60cm which has a breaking tension of 20N. The string is whirled in a vertical circle the axis of rotation at a height of 100cm from the ground.
   i) What is the angular velocity where the string is most likely to break?
   ii) How long will it take before the stone hits the ground?
   iii) Where the stone hit the ground

Solution

i) The string breaks when $T_{max} = \frac{m\,v^2}{r} + mg$

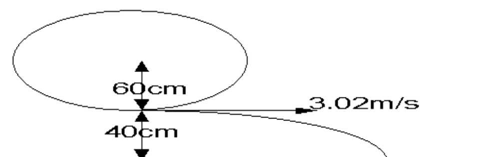
$20 = 0.8(9.81 + \frac{v^2}{0.6})$

$v = 3.02ms^{-1}$

But $v = r\,\omega$

$\omega = \frac{3.02}{0.6} = 5.03rads^{-1}$

ii)



$y = -40cm$ (below the point of projection)

$-0.4 = 3.02tsin0 - ½\,x9.81t^2$

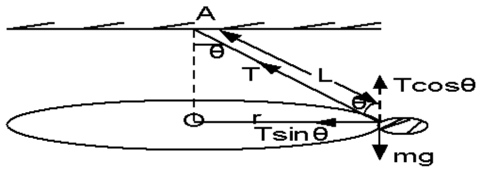
$t = 0.286s$

iii) Horizontal range

$x = utcos\theta = 3.02x0.285cos0 = 0.86m$

7.1.4: MOTION IN A HORIZONTAL CIRCLE [CONICAL PENDULUM]

Consider a body of mass , m tied to a string of length L whirled in a horizontal circle of radius r at a constant speed V



If the string is fixed at A and the centre O of the circle is directly below O, the horizontal components of the tension provides the necessary centripetal force.

$(\rightarrow)\ Tsin\theta = \frac{m\,v^2}{r}$ ----------- (1)

$(\uparrow)\ Tcos\theta = mg$ ----------- (2)

(1) ÷ (2): $tan\theta = \frac{v^2}{r\,g}$

$\boxed{v^2 = rgtan\theta}$ -------- (3)

but also $sin\theta = \frac{r}{L}$

r= $Lsin\theta$

and $v = r\,\omega$

put into equation (3)

$(r\,\omega)^2 = rgtan\theta$

$\omega^2 = \frac{g\tan\theta}{r}$

But $r = Lsin\theta$

$\omega^2 = \frac{g\tan\theta}{L\sin\theta} = \frac{g}{L\sin\theta}\,{sin\theta}/{cos\theta}$

$\boxed{\omega = \sqrt{\frac{g}{Lcos\theta}}}$ ---------- (4)

Also $T = \frac{2\,\pi}{\omega} = \frac{2\,\pi}{\sqrt{\frac{g}{Lcos\theta}}}$

$\boxed{T = 2\pi\sqrt{\frac{Lcos\theta}{g}}}$

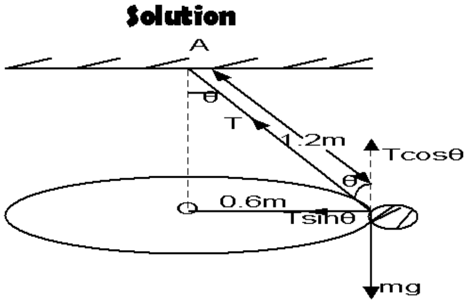
Explain why a mass attached to a string rotating at a constant speed in a horizontal circle will fly off at a tangent if the string breaks

- When a mass is whirled in a horizontal circle, the horizontal component of the tension ($Tsin\theta$) provides the necessary centripetal force which keeps the body moving in a circle without falling off.
- When the string breaks, the mass will not have any centripetal force and will continue in a straight line along the tangent.

Example

1. A stone 0.5kg is tied to one end of a string 1.2m long and whirled in a horizontal circle of diameter 1.2m. Calculate;
   i) The length in the string ii) The angular velocity iii) The period of motion

Solution



i) $(\uparrow)Tcos\theta = 0.5gN$ ---(1)

But $sin\theta = \frac{0.6}{1.2}$ ∴ θ= 30°

put into: (1) $Tcos30 = 0.5x9.81$

$T = 5.60N$

ii) Angular velocity

$\omega = \sqrt{\frac{g}{Lcos\theta}}$

$\omega = \sqrt{\frac{9.81}{1.2cos30}}$

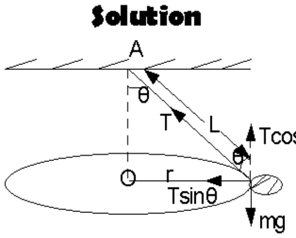
ω =3.07rads^{-1}

iii)Period , $T = \frac{2\,\pi}{\omega}$

$T = \frac{2x\frac{22}{7}}{3.07} = 2.05s$

2. A body of mass 4kg is moving with a uniform speed 5ms^{-1} in a horizontal circle of radius 0.3m, find:
   i) The angle the string makes with the vertical
   ii) The tension on the string

Solution



$(\rightarrow)\ Tsin\theta = \frac{m\,v^2}{r}$ ....[1]

$(\uparrow)Tcos\theta = mg$ ……….. [2]

[1]÷[2] $tan\theta = \frac{v^2}{r\,g}$

$\theta = tan^{-1}\frac{v^2}{r\,g}$

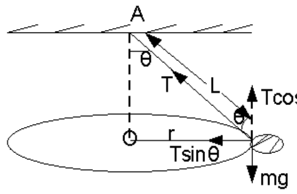
$\theta = tan^{-1}\left(\frac{5^2}{0.3x9.81}\right) = 83.3°$

ii) $Tcos\theta = mg$

$T = \frac{4x9.81}{Cos\ 83.3} = 336.33N$

3. The period of oscillation of a conical pendulum is 2s. If the string makes an angle of 60° with the vertical at the point of suspension, Calculate;
   i) The length of the string ii) The velocity of the mass

Solution



$\theta = 60°$

$sin60° = \frac{r}{l}$

r= $l$sin60°------------- (1)

(↑) $Tcos\theta = mg$

(→) $Tsin\theta = \frac{m v^2}{r}$

$Tan\theta = \frac{v^2}{rg}$

$v^2 = rgtan60$ ------- (2)

Also $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = 3.14$rads^{-1}

But $v = r\omega = 3.14r$

Put into equation (2)

$v^2 = rgtan60$

$(3.14r)^2 = rgtan60$

$r = \frac{g\ tan60}{314^2}$

put into equation (1)

$r = Lsin60$

$\frac{g\ tan60}{314^2} = lsin60$

$L = \frac{g\ tan60}{3.14^2 sin60} = 1.986m$

OR

$T = 2\pi\sqrt{\frac{Lcos\theta}{g}}$

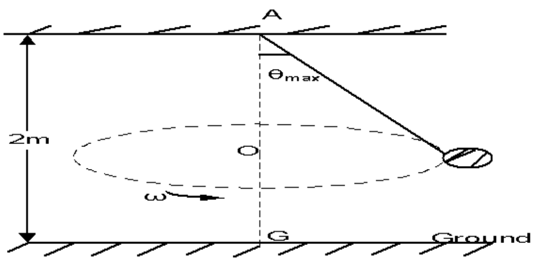
$L = \frac{T^2 g}{4\pi^2 cos\theta}$

$L = \frac{2^2 x9.81}{4\left(\frac{22}{7}\right)^2 cos60} = 1.986m$

$v = r\omega$

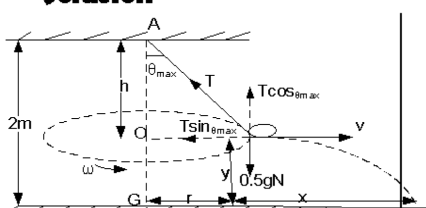
$v = \frac{2\pi}{T}r = \frac{2\pi}{2}\ l$sin60°=5.4ms^{-1}

4. Stone of mass 0.5kg is tied to one end of the string 1m long. The point of suspension of the string is 2m above the ground. The stone is whirled in the horizontal circle with increasing angular velocity. The string will break when the tension in it is 12.5N and the angle θ is to the maximum (θ_{max})as shown in the figure below;



i) Calculate the angle θ_{max}
ii) Calculate the angular velocity of the stone when the string breaks
iii) How far from the point G on the ground will the stone hit the ground
iv) What will be the speed of the stone when it hits the ground

Solution



(↑) T$cos\theta_{max}$= $0.5gN$

$cos\theta_{max} = \frac{0.5x9.81}{12.5}$

θ_{max}= 66.9°

(→) Tsinθ = $m\omega^2$r

Also sinθ = $\frac{r}{l}$

T$\frac{r}{1} = m\omega^2$r

T=m ω^2

$\omega^2 = \frac{12.5}{0.5}$ rads^{-1}

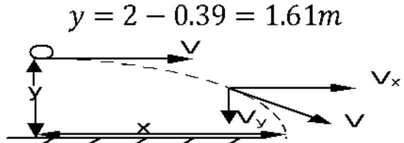
ω = 5rads^{-1}

$Cos\theta_{max} = \frac{h}{1}$

$h = cos66.9 = 0.39m$

$y + h = 2$

$y = 2 - 0.39 = 1.61m$



Using $y = utsin\theta - ½\ gt^2$

$y = -1.61$ below the point of projection

$-1.61 = ut\ sin0 - ½\ x9.81t^2$

$-1.61 = -½\ x9.81t^2$

Horizontal distance

$x = vcos\theta t$

$x = vcos0x0.57 = 0.57v$

but $v = \omega r$

x =0.57 ωr

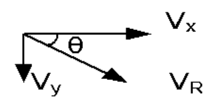
$x = 0.57x5xsin66.9$

where $sin66.9 = \frac{r}{l}$

$x = 2.63m$

$\therefore G = r + x$

$G = 2.62 + sin66.9 = 3.54m$



v_x is constant

v_x =ucosθt

$v_x = vcos0x0.57 = 0.57v$

$v_x = 0.57\omega r = 0.57x5xsin66.9$

v_x =4.599ms^{-1}

$v_y = usin\theta + gt$

$v_y = usin0 + 9.91x0.57$

$v_y = 5.592ms^{-1}$

$v_R = \sqrt{V_X^2 + V_y^2}$

$v_R = \sqrt{4.599^2 + 5.592^2}$ =7.24ms^{-1}

θ =tan$^{-1}\frac{5.592}{4.599}$

The speed as it hits the ground is 7.24ms^{-1}.

EXERCISE:16

1. A stone of mass 500g is attached to string of length 50cm which will break when the tension in it exceeds 20N. The string is whirled in a vertical circle the axis of rotation at a height of 100cm from the ground.
   i) What is the angular velocity where the string is most likely to break?

ii) Where will the stone hit the ground **An(7.8rad**s^{-1}, $1.25m$)

3. A bucket of water is swung in a vertical circle of radius 64.0m in such a way that the bucket is upside down when it is at the top of the circle. What is the minimum speed that the bucket ,may at this point if the water is to remain in it. **An[25.06ms^{-1}]**
4. An aero plane loop a path in a vertical circle of radius 200m, with a speed of $40ms^{-1}$ at the top of the path. The pilot has a mass of 80kg. what is the tension in the strap holding the pilot into his seat when he is at the top of the path **An[60N]**
5. An astronaut loop a path in a horizontal circle of radius 5m. if he can withstand a maximum acceleration of 78.5ms^{-2} .What is the maximum angular velocity at which the astronaut can remain conscious **An[** $3.96rads^{-1}$**]**
6. A body of mass 20kg is whirled in a horizontal circle using an inelastic string which has a breaking force of 400N. If the breaking speed is at 9ms^{-1}. Calculate the angle which the string makes with the horizontal at the point of breaking. **An(θ=29.3°).**
7. A particle of mass 0.2kg is attached to one end of a light inextensible string of length 50cm. The particle moves in a horizontal circle with an angular velocity of 5.0rads^{-1} with the string inclined at θ to the vertical. Find the value of θ. **An(37°)**
8. A particle of mass 0.25kg is attached to one end of a light in extensible string of length 3.0m. The particle moves in a horizontal circle and the string sweeps out the surface of a cone. The maximum tension that the string can sustain is 12N. Find the maximum angular velocity of the particle. **An[4rads^{-1}].**
9. A particle of mass 0.30kg moves with an angular velocity Of 10rads^{-1} in a horizontal circle of radius 20cm inside a smooth hemispherical bowl. Find the reaction of the bowl on the particle and the radius of the bowl. **An[6.7N, 22cm]**
10. A child of mass 20kg sits on a stool tied to the end of an inextensible string 5m long, the other end of the string being tied to a fixed point. The child is whirled in a horizontal circle of radius 3m with a child not touching ground.
    i) Calculate the tension on the string
    ii) Calculate the speed of the child as it moves around the circle. **An[245.25N, 4.695ms^{-1}]**

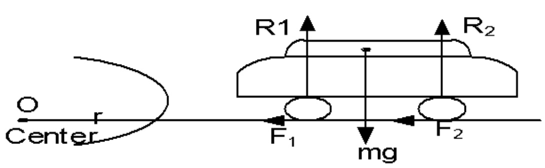
7.1.5: MOTION OF A CAR ROUND A FLAT HORIZONTAL TRACK [NEGOTIATING A BEND]

Consider a car of mass m moving round a circular horizontal arc of radius r with a speed v

A) Skidding of the car

Skidding is the failure of a vehicle to negotiate a curve as a result of having a centripetal force less than the centrifugal force and the car goes off the track or moves away from the centre of the circle.

Consider a car of mass m taking a flat curve of radius r at a speed v. F_1 and F_2 are the frictional forces due to the inner tyre and outer tyre respectively. R_1 and R_2 are the normal reactions due to inner and outer tyres respectively.



$(\uparrow): R_1 + R_2 = mg$--------- (1)

$(\rightarrow): F_1 + F_2 = \frac{mv^2}{r}$ -----(2)

The frictional forces F_1 and F_2 provide the necessary centripetal force

But $F_1 = \mu R_1$, $F_2 = \mu R_2$

$\mu R_1 + \mu R_2 = \frac{mv^2}{r}$

$\mu (R_1 + R_2) = \frac{mv^2}{r}$ --------------------- (3)

Put equation (1) into equation (3)

$\mu mg = \frac{mv^2}{r}$

$v^2 = rg\mu$

The maximum speed with which no skidding occurs is given by

$$\boxed{v_{max} = \sqrt{\mu rg}}$$

For no skidding

$\mu \geq \frac{v^2}{rg}$ Or $v^2 \leq \mu rg$

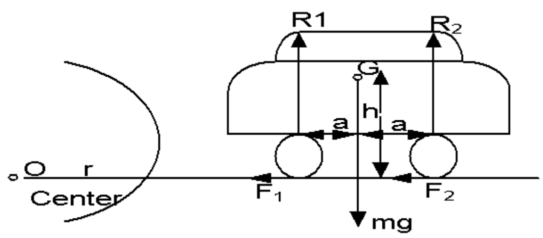
Conditions for no skidding/side slips

For a car to go round a bend successfully without skidding then:

1- The speed should not exceed $(\mu rg)^{\frac{1}{2}}$ or $[v \le \sqrt{\mu rg}]$
2- The radius of the bend should be made big
3- Coefficient of friction should be increased
4- Centre of gravity should be low

B) Overturning/toppling of a car

Consider a car of mass m moving around a horizontal (flat)circular bend of radius r at speed v. let the height of the centre of gravity above the track be "h" and the distance between the wheels be "2a".



(↑): $R_1 + R_2$ =mg----- (1)

(→): $F_1 + F_2 = \frac{mv^2}{r}$ -----(2)

Taking moments about G

Clockwise moments =anticlockwise moments

$F_1.h + F_2.h + R_1.a = R_2.a$

$(F_1 + F_2)h + R_1 a = R_2.a$ ------------- (3)

Put equation 2 into equation 3

$\frac{mv^2}{r}.h + R_1 a = R_2.a$

$\frac{mv^2}{r}.\frac{h}{a} = (R_2 - R_1)$---------------------[4]

Equation 1+Equation 4

$R_1 + R_2 + \frac{mv^2}{r}.\frac{h}{a} = (R_2 - R_1) + mg$

$2R_1 = mg - \frac{mv^2 h}{ra}$

$R_1 = \frac{m}{2}\left(g - \frac{v^2 h}{ra}\right)$---------------------- (5)

A car just topples or upsets when $R_1 = 0$

$\frac{m}{2}\left(g - \frac{v^2 h}{ra}\right) = 0$

$g = \frac{v^2 h}{ra}$

$v_{max} = \sqrt{\frac{rag}{h}}$

Note

R_1 is the reaction of the inner tyre

- When $R_1>0$: The wheels in the inner side of the curve is in contact with the ground
- When $R_1=0$: The wheels in the inner side of the curve are at the point of loosing contact with the ground
- When $R_1<0$: The inner wheels have lost contact with the ground and the vehicle has over turned

Ways prevent toppling/overturning

i) Reduce the speed when negotiating a corner ($v^2 \le \frac{rag}{h}$)

ii) Increase radius of a corner ($r > \frac{v^2 h}{ra}$)

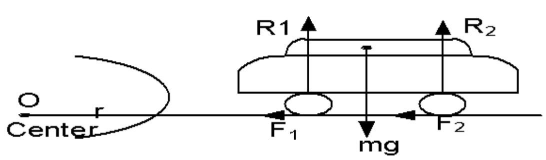
iii) The distance between the tyres should be made big ($a > \frac{v^2 h}{ra}$)

iv) Reduce distance from the ground to the centre of gravity (h) or C.O.G of the car should be low ($h < \frac{rag}{v^2}$)

EXAMPLE

1. A car of mass 1000kg goes round a bend of radius 100m at a speed of 50km/hr without skidding. Determine the coefficient of friction between the tyres and the road surface

Solution



(↑): $R_1 + R_2 = mg$----- (1)

(→): $F_1 + F_2 = \frac{mv^2}{r}$

$\mu(R_1 + R_2) = \frac{mv^2}{r}$-----------------------[2]

Put equation (1) and equation 2: μ mg= $\frac{mv^2}{r}$

$\mu = \frac{v^2}{rg} = \frac{\left(\frac{50x1000}{3600}\right)^2}{100x9.81} = 0.1965$

MOTION OF A CAR ON A BANKED TRACK

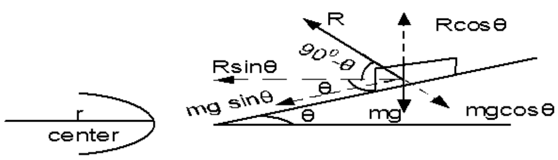
Definition :Banking a track is the building of a track round a corner with the outer edge raised above the inner one.

Banking ensures that only the horizontal component of normal reaction contributes towards the centripetal force.

Banking also enables the car to go round a bend at a higher speed for the same radius compared to a flat track.

A) NO SIDE SLIPP [No frictional force]

Consider a car of mass m negotiating a banked track at a speed v and radius of the bend is r.



(↑) :$R\cos\theta = mg$ ------- (1)

(→):$R\sin\theta = \frac{mv^2}{r}$----(2)

(2) ÷ (1): $\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{rmg}$

$\tan\theta = \frac{v^2}{rg}$

$v^2 = rg\tan\theta$

θ is the angle of banking and v is the designed speed of the banked track.

Examples

1. A racing car of mass 1000kg moves around a banked track at a constant speed of 108km/hr, the radius of the track is 100m. Calculate the angle of banking and the total reaction at the tyres.

Solution

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left[\frac{\left(\frac{108x1000}{3600}\right)^2}{100x9.81}\right]$ =42.5°

Resolving vertically:$R\cos\theta = mg$

$R = \frac{1000x9.81}{\cos 42.5} = 13305N$

Exercise :17

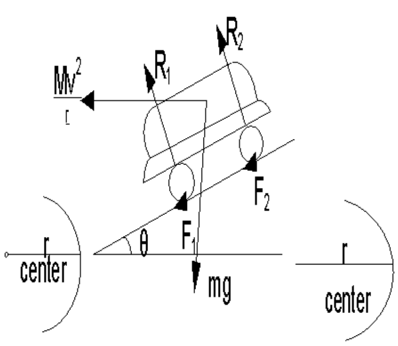
1. A road banked at 10° goes round a bend of radius 70m. At what speed can a car travel round the bend without tending to side slip. **An[11ms⁻¹]**
2. A car travels round a bend of radius 400m on a road which is banked at an angleθ to the horizontal. If the car has no tendency to skid when traveling at 35ms^{-1}, find the value of θ **An[17.34°]**
3. A driver has to drive a car in a horizontal circular path of radius 105m around a bend that is banked at 45° to the horizontal. The driver finds that he must drive with a speed of at least 21ms^{-1} if he is to avoid slipping sideways. Find the coefficient of friction between the tyres of the car and road **An[0.4]**

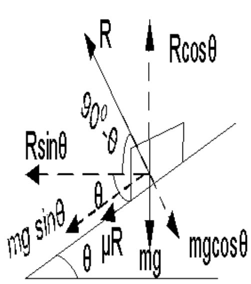
B) SKIDDING/SLIDE SLIPP

The frictional force must be there whose direction depends on the speed of the car.

(i) MAXIMUM SPEED/GREATEST SPEED

If the car is moving at speed v, greater than the designed speed v, the force Rsinθ is enough to provide the necessary centripetal force. The car will tend to slid outwards from the circular path, the frictional force would therefore oppose their tendency up to the maximum value .





(↑): $R\cos\theta = mg + \mu R\sin\theta$

$R(\cos\theta - \mu\sin\theta) = mg$ ------- (1)

(→): $R\sin\theta + \mu R\cos\theta = \frac{mv^2}{r}$

$R(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}$ ------ (2)

(2)÷ (1): $\frac{R(\sin\theta + \mu\cos\theta)}{R(\cos\theta - \mu\sin\theta)} = \frac{mv^2}{rmg}$

$\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} = \frac{v^2}{r}g$

$v_{max}^2 = rg\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$

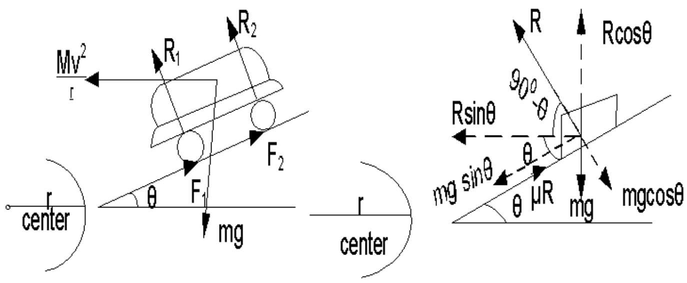
Or divide the right hand side by cosθ

$v_{max}^2 = rg\left[\frac{(\tan\theta + \mu)}{(1 - \mu\tan\theta)}\right]$

(ii) MINIMUM SPEED/LEAST SPEED

If the speed v, is less than the designed speed v the component of the reaction $R\sin\theta$ produces an acceleration greater than the centripetal acceleration $(\frac{v^2}{r})$ which is required to keep the car on circular path.

The car tends to slip down the banked track and this tendency is opposed by the frictional force acting upwards.



$(\uparrow): R\cos\theta + \mu R\sin\theta = mg$

$R(\cos\theta + \mu\sin\theta) = mg$ ------------ (1)

$(\rightarrow): R\sin\theta - \mu R\sin\theta = \frac{mv^2}{r}$

$R(\sin\theta - \mu\cos\theta) = \frac{mv^2}{r}$------------ (2)

(2)÷ (1): $\frac{(Sin\theta - \mu Cos\theta)}{(Cos\theta + \mu Sin\theta)} = \frac{v_{min}^2}{rg}$

Divide the right hand side by cosθ

$v_{min}^2 = rg\left[\frac{(tan\theta - \mu)}{(1 + \mu tan\theta)}\right]$

Example

1. A car travels round a bend which is banked at 22°. If the radius of the curve is 62.5m and the coefficient of friction between the road surface and tyres of the car is 0.3, calculate the maximum and minimum speed at which the car can negotiate the bend without skidding.

Solution

$v_{max}^2 = rg\left[\frac{(tan\theta + \mu)}{(1 - \mu tan\theta)}\right]$

$v_{max} = \left[62.5x9.81\left(\frac{tan22 + 0.3}{1 - 0.3tan22}\right)\right]^{\frac{1}{2}}$ =22.15ms⁻¹

$v_{min}^2 = rg\left[\frac{(tan\theta - \mu)}{(1 + \mu tan\theta)}\right]$

$v_{min} = \left[62.5x9.81\left(\frac{tan22 - 0.3}{1 + 0.3tan22}\right)\right]^{\frac{1}{2}}$ =7.54ms⁻¹

2. On a level race track, a car just goes round a bend of radius 80m at a speed of 20ms⁻¹ without skidding. At what angle must the track be banked so that a speed of 30ms⁻¹ can just be reached without skidding, the coefficient of friction being the same in both cases.

Solution

Case I: of a level track

For no skidding $V_{max} = \sqrt{\mu rg}$

$20^2 = \mu x80x9.81$

$\mu = 0.51$

Case II: on a banked track

$v_{max}^2 = rg\left[\frac{(tan\theta + \mu)}{(1 - \mu tan\theta)}\right]$

$30^2 = 80x9.81\left[\frac{(tan\theta + \mu)}{(1 - \mu tan\theta)}\right]$

$\frac{900}{80x9.81} = \frac{(tan\theta + 0.51)}{(1 - 0.51tan\theta)}$

$tan\theta = \frac{0.6368}{1.58468}$

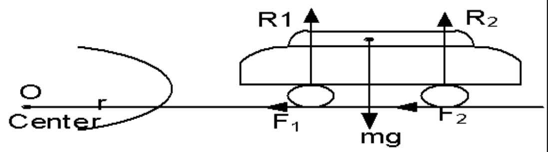
$\theta = 21.89°$

EXERCISE:18

1. A racing car of mass 2 tonnes is moving at a speed of 5ms⁻¹ round a circular path. If the radius of the track is 100m. calculate;
   i) Angle of inclination of the track to the horizontal if the car does not tend to side slip
   ii) The reaction to the wheel if it's assumed to be normal to the track. **An[1.5°, 19606.7N]**
2. A car travels round a bend banked at an angle of 22.6°. if the radius of curvature of the bend is 62.5m and the coefficient of friction between the tyres of the car and the road surface is 0.3. Calculate the maximum and minimum speed at which the car negotiates the bend without skidding. **An [22.38ms⁻¹, 7.96ms⁻¹]**
3. A car moves in a horizontal circle of radius 140cm around a banked corner of a track. The maximum speed with which the car can be driven around the corner without slipping occurring is $42ms^{-1}$. If the coefficient of friction between the tyres of the car and the surface of the track is 0.3. find the angle of banking **An[71.1°]**

Question: Explain why a car travels at a higher speed round a banked track without skidding unlike the flat tracks of the same radius.

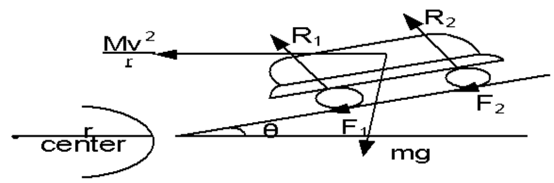
Solution



Along a circular arc on a horizontal road the frictional force provides the centripetal force

$F_{max} = \frac{mv^2}{r} = \mu R$

At a higher speed, the frictional force is not sufficient enough to provide the necessary centripetal force and skidding would occur.



On a banked track the centripetal force is provided by both the horizontal component of normal reaction R and component of the frictional force. $F_c = F\cos\theta + R\sin\theta = \frac{mv^2}{r}$

For $0° < \theta < 90°$, $\mu\cos\theta + \sin\theta > \mu$ therefore $V_1 < V_2$

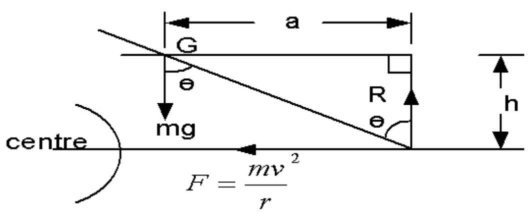
This is enough to keep the car on the track even at high speed.

7.1.7: MOTION OF A CYCLIST ROUND A BEND

A cyclist must bend towards the centre while travelling round the bend to avoid toppling. When the cyclist bends, the weight creates a couple which opposes the turning effect of the centrifugal forces.

Consider the total mass of the cyclist and his bike to be m round the circle of radius r at a speed v.

A) No skidding



(↑): $R = mg$ ---------- (1)

(→): $\mu R = \frac{mv^2}{r}$-------------------- (2)

Put 1 into 2: $\mu mg = \frac{mv^2}{r}$

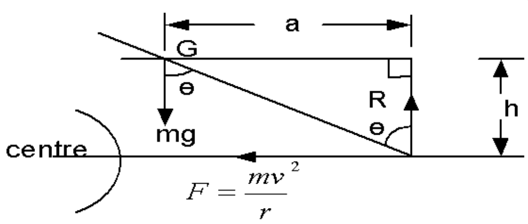
$v^2 = \mu rg$

V is the max speed at which a cyclist negotiates a bend of radius r without skidding

For no skidding : $v^2 \leq \mu rg$

B) No toppling/over turning

The force G has a moment about the centre of gravity G(F.h) which tends to turn the rider out.



(↑) $R = mg$

Taking moment about G: $\frac{mv^2}{r}.h = R.a$

$\frac{a}{h} = \frac{\frac{mv^2}{r}}{R}$

But $\tan\theta = \frac{a}{h}$

$\tan\theta = \frac{\frac{mv^2}{r}}{mg}$

$\boxed{v^2 = rg\tan\theta}$

v is the speed at which a cyclist can negotiate a corner without toppling

For no toppling $v^2 \leq rg\tan\theta$

Why it is necessary for a bicycle rider moving round a circular path to lean towards a center of the path

When a rider moves round a circular path, the frictional force provides the centripetal force. The frictional force has a moment about the centre of gravity of the rider, the rider therefore tends to fall off from the centre of the path if this moment is not counter balanced. The rider therefore leans toward the center of the path so that his reaction provides a moment about the center of gravity , which counter balances the moment due to friction.

UNEB 2014No1

(b) (i) Define angular velocity. (01mark)

(ii) satellite is revolving around the earth in a circular orbit at an altitude of $6\,x10^5m$ where the acceleration due to gravity is $9.4ms^{-2}$. Assuming that the earth is spherical, calculate the period of the satellite. **An**$[5.42x10^3s]$ (03marks)

UNEB2013No3

(b) Show that the centripetal acceleration of an object moving with constant speed, v, in a circle of radius, r, is $\frac{v^2}{r}$ (04marks)

(c) A car of mass 1000kg moves round a banked track at a constant speed of 108kmh^{-1}. Assuming the total reaction at the wheels is normal to the track, and the radius of curvature of the track is 100m, calculate the;

(i) Angle of inclination of the track to the horizontal. **An[42.5°]** (04marks)

(ii) Reaction at the wheels **An[13305N]** (02marks)

UNEB 2012 No3

a) Explain what is meant by centripetal force (2mks)

b) i) Derive an expression for the centripetal force acting on a body of mass m moving in a circular path of radius r (6mrk)

ii) A body moving in a circular path of radius 0.5m makes 40 revolutions per second. Find the centripetal force if the mass is 1kg (3mk)

c) Explain the following;

i) a mass attached to a string rotating at a constant speed in a horizontal circle will fly off at a tangent if the string break (02mk)

ii) a cosmonaut in a satellite which is in a free circular orbit around the earth experiences the sensation of weightlessness even though there is influence of gravitation field of the earth.

Solution

b) ii f=40revs^{-1} r=50m m=1kg

$\omega = 2\pi f = 2\text{x}\frac{22}{7}\text{x}40 = 251.43\text{rads}^{-1}$

F=mω^2r=1(251.43)^{2}x50= 3.161x10^2N

UNEB 2011 No1

a) Define the following terms

i) Uniform acceleration (1mk)

ii) Angular velocity (1mk)

b) i) what is meant by banking of a track

(ii) Derive an expression for the angle of banking θ for a car of mass, m moving at a speed, v around banked track of radius r. (4mk)

c) A bob of mass, m tied to an inelastic thread of length L and whirled with a constant speed in a vertical circle

i) With the aid of a sketch diagram, explain the variation of tension in the string along the circle (5mk)

ii) If the string breaks at one point along the circle state the most likely position and explain the subsequent motion of the bob. [2mk]

UNEB 2007 No1

d) Explain why the maximum speed of a car on a banked road is higher than that on an unbanked road.

e) A small bob of mass 0.20kg is suspended by an inextensible string of length 0.8m. The bob is then rotated in a horizontal circle of radius 0.4m. find the

i) linear speed of the bob (3mk)

ii) tension in the string (2mk)

UNEB 2005 No4

a) i) Define angular velocity (1mk)

ii) Derive an expression for the force F on a particle of mass m, moving with angular velocity ω in a circle of radius r.

b) A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1m above he ground. The angular speed is gradually increased until the string breaks.

i) In what position is the string most likely to break? Explain.

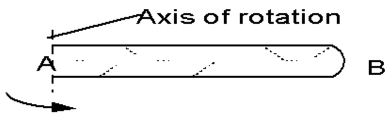
ii) At what angular speed will the string break **An [7.78rads^{-1}, 1.24m]**

iii) Find the position where the stone hits the ground when the string breaks

c) Explain briefly the action of a centrifuge

Solution

Action of a centrifuge



A centrifuge is used to separate substances of different densities e.g. milk and fat by whirling in a horizontal circle at a high speed.

The mixture placed in a tube and the tube is rotated in a horizontal circle. The liquid pressure at the closed end B is more than that at the open end A. this sets up a pressure gradient along the tube. This pressure gradient creates a large centripetal force that causes matter of small density to move inwards while that of higher density to move away from the centre when rotation stops, the tube is placed in a vertical position and the less dense substance comes to the top which are then separated from the mixture.

UNEB 2004 No2

a) Define the term angular velocity (1mk)

b) A car of mass m, travels round a circular track of radius, r with a velocity v.

i) Sketch a diagram to show the forces acting on the car (2mks)

ii) Show that the car does not overturn if $v^2 < \frac{arg}{2h}$, where a is the distance between the wheel, h is the height of the C.O.G above the ground and g is the acceleration due to gravity

c) A pendulum of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. the bob moves in a horizontal circle with the string inclined at 30° to the vertical . Calculate;

i) The tension in the string (2mk)

ii) The period of the motion **An[2.27N, 2.04s]** (4mk)

UNEB 2003 No2

a) Define the following terms

i) Angular velocity (1mk)

ii) Centripetal acceleration (1mk)

b) i) Explain why a racing car travels faster on a banked track than one which is flat of the same radius of curvature. (4mk)

ii) Derive an expression for the speed with which a car can negotiate a bend on a banked track without skidding (3mk)

UNEB 2002 No1

d) The period of oscillation of a conical pendulum is 2.0s. if the string makes an angel 60° to the vertical at the point of suspension, calculate the

i) Vertical height of the point of suspension above the circle (3mk)

ii) Length of the string (1mk)

iii) Velocity of the mass attached to the string (3mk)

An[0.995m, 1.99m, 5.41ms^{-1}]

UNEB 2002 No2

b i) Derive an expression for the speed of a body moving uniformly in a circular path (3mk)

ii) Explain why a force is necessary to maintain a body moving with a constant speed in a circular path.

c) A small mass attached to a string suspended from a fixed point moves in a circular path at a constant speed in a horizontal plane.

i) Draw a diagram showing the forces acting on the mass (1mk)

ii) Derive an equation showing how the angle of inclination of the string depend on the speed of the mass and the radius of the circular path (3mk).